CH 2 HOMEWORK ANSWERS

2.4

We know that

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

The signals $x[n]$ and $y[n]$ are as shown in Figure S2.4. From this figure, we see that the above summation reduces to

$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$

This gives

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{otherwise} \end{cases}$$

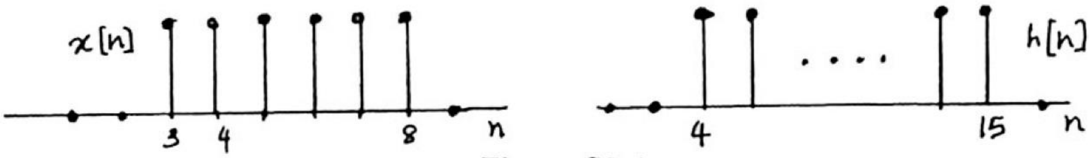

Figure S2.4

2.8

Using the convolution integral,

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau.$$

Given that $h(t) = \delta(t+2) + 2\delta(t+1)$, the above integral reduces to

$$x(t) * y(t) = x(t+2) + 2x(t+1)$$

The signals $x(t+2)$ and $2x(t+1)$ are plotted in Figure S2.8.

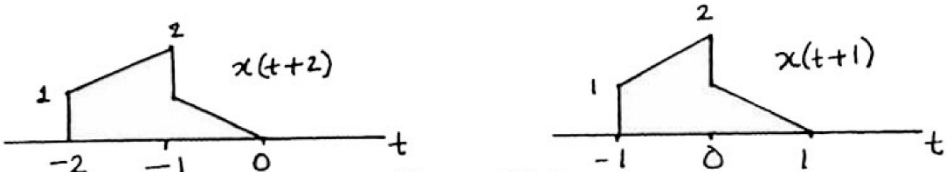

Figure S2.8

Using these plots, we can easily show that

$$y(t) = \begin{cases} t+3, & -2 < t \le -1 \\ t+4, & -1 < t \le 0 \\ 2-2t, & 0 < t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

2.14

(a) We first determine if $h_1(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_1(\tau)| d\tau = \int_{0}^{\infty} e^{-t} d\tau = 1$$

Therefore, $h_1(t)$ is the impulse response of a stable LTI system.

(b) We determine if $h_2(t)$ is absolutely integrable as follows

$$\int_{-\infty}^{\infty} |h_2(\tau)| d\tau = \int_{0}^{\infty} e^{-t} |\cos(2t)| d\tau$$

This integral is clearly finite-valued because $e^{-t}|\cos(2t)|$ is an exponentially decaying function in the range $0 \le t \le \infty$. Therefore, $h_2(t)$ is the impulse response of a stable LTI system.

2.48

(a) Note that

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau = \int_{-\infty}^{t-2} e^{-(t-2-\tau')} x(\tau') d\tau'.$$

Therefore,

$$h(t) = e^{-(t-2)} u(t-2).$$

(b) We have

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \\ &= \int_{2}^{\infty} e^{-(\tau-2)} [u(t-\tau+1) - u(t-\tau-2) \end{aligned}$$

$h(\tau)$ and $x(t-\tau)$ are as shown in the figure below.

Using this figure, we may write

$$y(t) = \begin{cases} 0, & t < 1 \\ \int_{2}^{t+1} e^{-(\tau-2)} d\tau = 1 - e^{-(t-1)}, & 1 < t < 4 \\ \int_{t-2}^{t+1} e^{-(\tau-2)} d\tau = e^{-(t-4)}[1 - e^{-3}], & t > 4 \end{cases}$$

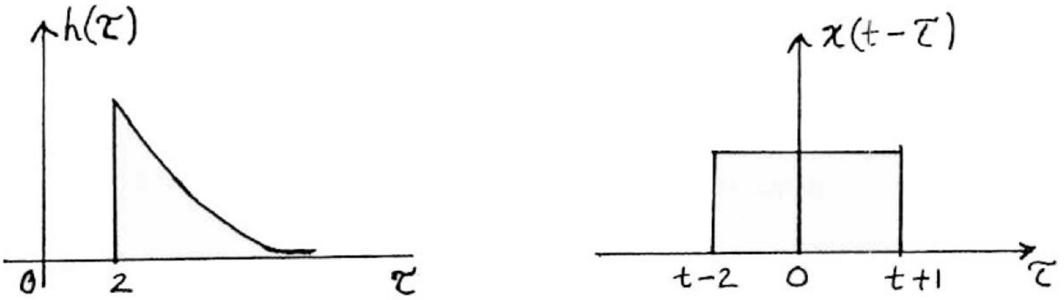

Figure S2.40

2.50

We have

$$y(t) = x(t) * h(t) = \int_{-0.5}^{0.5} e^{j\omega_0(t-\tau)} d\tau.$$

Therefore,

$$y(0) = \int_{-0.5}^{0.5} e^{-j\omega_0\tau} d\tau = \frac{2}{\omega_0}\sin(\omega_0/2).$$

(a) If $\omega_0 = 2\pi$, then $y(0) = 0$.

(b) Clearly, our answer to part (a) is not unique. Any $\omega_0 = 2k\pi$, $k \in \mathcal{I}$ and $k \neq 0$ will suffice.

(d) (i) Taking

$$y(t) = \sum_r \alpha u_r(t)$$

we get

$$\sum_r [\alpha_r u_{r+2}(t) + 3\alpha_r u_{r+1}(t) + 2\alpha_r u_r] = \delta(t)$$

This implies that $r_{max} = -2$ and $\alpha_{-2} = 1$. Therefore, $h(0+) = 0$ and $h'(0^+) = 1$ constitute the initial conditions. Now,

$$s^3 + 3s + 2 = 0 \qquad \Rightarrow \qquad s = -2, s = -1.$$

Therefore,

$$h(t) = Ae^{-2}t + Be^{-t},\ t \geq 0.$$

Applying initial conditions, we get $A = -1$, $B = 1$. Therefore,

$$h(t) = (e^{-t} - e^{-2t})u_{-1}(t).$$

(ii) The initial conditions are $h(0^+) = 0$ and $h'(0^+)1$, Also, $s = -1 \pm j$. Therefore

$$h(t) = [e^{-t}\sin t]u_{-1}(t).$$

(e) From part (c), if $M \geq N$, then $\sum_{k=0}^{M} b_k \frac{d^k h_b(t)}{dt^k}$ will contain singularity terms at $t = 0$ This implies that

$$h(t) = \sum_r \alpha_r u_r(t) + \cdots.$$

(f) (i) Now,

$$\sum_r \alpha_r u_{r+1}(t) + 2\sum \alpha_r u_r = 3u_1(t) + u_0(t).$$

Therefore, $r_{max} = 0$. Also

$$\alpha_0 u_1(t) + \alpha_{-1}u_0(t) + 2\alpha_0 u_0(t) = 3u_1(t) + u_0(t).$$

This gives $\alpha_0 = 3$ and $\alpha_{-1} = -5$. The initial condition is $h(0^+) = -5$ and

$$h(t) = 3u_0(t) - 5e^{-2t}u_{-1}(t) = 3\delta(t) - 5e^{-2t}u(t).$$

(ii) Here, $\alpha_1 = 1$, $\alpha_0 = -3$, $\alpha_{-1} = 13$, $\alpha_{-2} = -44$. Therefore $h(0^+) = 13$ and $h'(0^+) = -44$ and

$$h(t) = u_1(t) - 3u_0(t) + 18e^{-3t}u_{-1}(t) - 5e^{-2t}u_{-1}(t).$$

2.68

(a) Realizing that $x_2[n] = y_1[n]$, we may eliminate these from the two given difference equations. This would give us

$$y_2[n] = -ay_2[n-1] + b_0x_1[n] + b_1x_1[n-1].$$

This is the same as the overall difference equation.

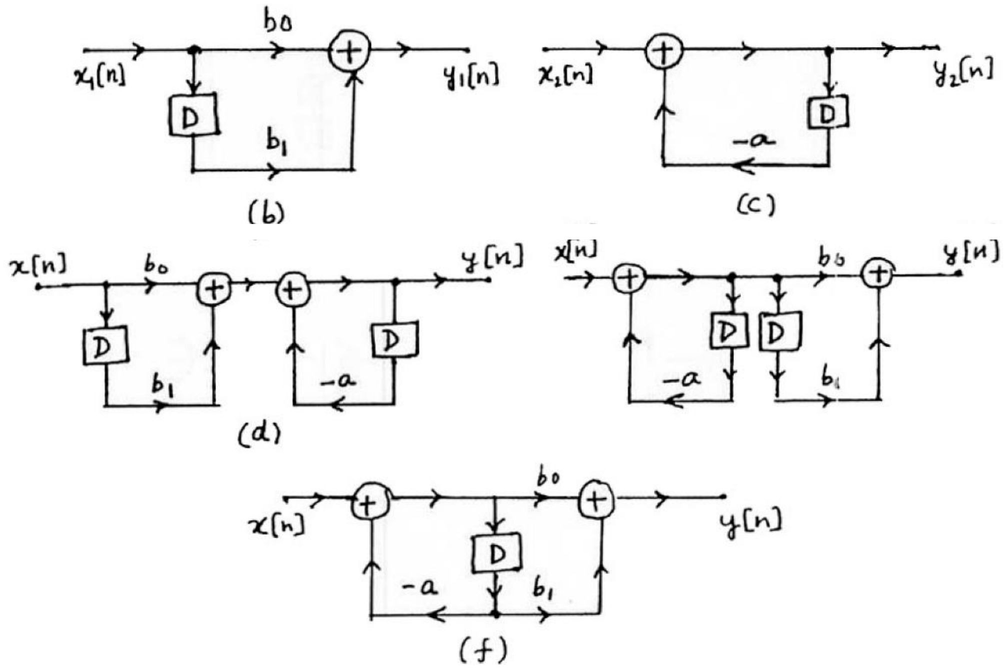

Figure S2.57

(b) The figures corresponding to the remaining parts of this problem are shown in the Figure S2.57.