CH7 ans

7.3

Given: $x(t) = 1 + \cos(200\pi t) + \sin(4000\pi t)$

The Nyquist rate will be equal to twice the highest frequency in the signal so:

$\omega_n = 2 \times w_{max}$

$\omega_n = 2 \times 4000\pi$

$\omega_n = 8000\pi$

Given: $x(t) = \dfrac{\sin(4000\pi t)}{\pi t}$

The Nyquist rate will be equal to twice the highest frequency in the signal so:

$\omega_n = 2 \times w_{max}$

$\omega_n = 2 \times 4000\pi$

$\omega_n = 8000\pi$

Given: $x(t) = \left(\dfrac{\sin(4000\pi t)}{\pi t}\right)^2$

The Nyquist rate will be equal to twice the highest frequency in the signal so:

$\omega_n = 2 \times w_{max}$

$\omega_n = 2 \times 8000\pi$

$\omega_n = 16000\pi$

7.6

The product $\omega(t)$ is given by,

$$\omega(t) = x_1(t)x_2(t)$$

Then, the Fourier transform of $\omega(t)$ is given by,

$$W(j\omega) = \frac{1}{2\pi}\left[X_1(j\omega) * X_2(j\omega)\right]$$

As given before, the maximum frequency of $x_1(t)$ equals ω_1 and that of $x_2(t)$ equals ω_2.

The maximum frequency of the two convoluted signals ω_M equals the sum of the two signals frequencies.

Then, ω_M is given by,

$$\omega_M = \omega_1 + \omega_2$$

Determine the Nyquist rate ω_N,

$$\omega_N = 2\omega_M$$

$$= 2(\omega_1 + \omega_2)$$

Determine the maximum sampling interval T,

$$T = \frac{2\pi}{\omega_N}$$

$$= \frac{2\pi}{2(\omega_1 + \omega_2)}$$

$$= \frac{\pi}{\omega_1 + \omega_2}$$

$$\boxed{T = \frac{\pi}{\omega_1 + \omega_2}}$$

7.8

Noting that,

$$x(t) = \left(\frac{\sin 50\pi t}{\pi t}\right)^2$$

$$= 2500 sinc^2(50\pi t) \ldots (1)$$

From the known Fourier transform,

$$\Delta\left(\frac{t}{\tau}\right) = \frac{\tau}{2} sinc^2\left(\frac{\omega\tau}{4}\right)$$

Using duality principle,

$$\frac{\tau}{2} sinc^2\left(\frac{t\tau}{4}\right) = 2\pi\Delta\left(\frac{\omega}{-\tau}\right)$$

$$sinc^2\left(\frac{t\tau}{4}\right) = \frac{4\pi}{\tau}\Delta\left(\frac{\omega}{\tau}\right) \ldots (2)$$

Comparing the sin functions in (1) and (2),

$$\frac{\tau}{4} = 50\pi$$

$$\tau = 200\pi$$

Calculating the Fourier transform $x(t)$,

$$X(j\omega) = 50\Delta\left(\frac{\omega}{200\pi}\right)$$

Sketching the spectrum of $X(j\omega)$,

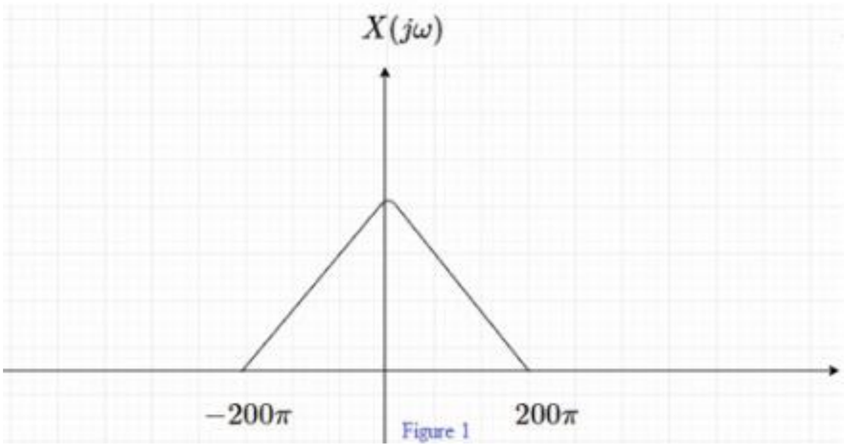

Figure 1

Calculating the Fourier transform of $g(t)$,

$$G(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$= \frac{1}{\frac{2\pi}{\omega_s}}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

$$= \frac{1}{\frac{2\pi}{150\pi}}\sum_{k=-\infty}^{\infty} X(j(\omega - k150))$$

$$= 75\sum_{k=-\infty}^{\infty} X(j(\omega - k150))$$

Sketching the spectrum of $G(j\omega)$,

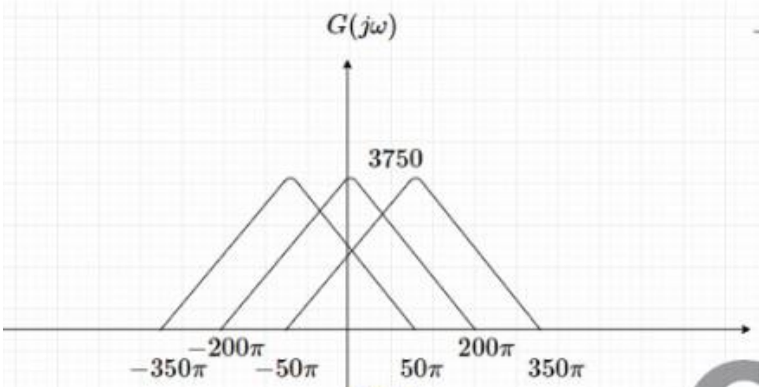

Figure 2

Referring to Figure 2, the maximum value of ω_0 for which it's guaranteed that $G(j\omega) = 75X(j\omega)$ for $|\omega| \leq \omega_0$,

$$\boxed{\omega_0 = 50\pi}$$

7.10

(a)

From the given expression of $x_d[n]$, the value of the sampling period T is given by,

$$T = 0.5 \text{ ms}$$

The signal $x_d[n]$ in the frequency domain is given by,

$$X_d(e^{j\Omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\Omega - 2\pi k)/T) \ldots (1)$$

The relation between the discrete frequency Ω and the continuous one ω is given by,

$$\Omega = \omega T$$

From 2, $X_d(e^{j\Omega})$ represents the shifting and summing of $X_c(j\Omega)$.

Since $X_d(e^{j\omega})$ is real then, $X_c(j\Omega)$ is real.

$$\boxed{X_c(j\Omega) \text{ is real}}$$

(b)

Rearrange 1,

$$\sum_{k=-\infty}^{\infty} X_c(j(\Omega - 2\pi k)/T) = T[X_d(e^{j\Omega})]$$

Since Max $\left\{X_d(e^{j\Omega})\right\}$ equals one, the maximum value of $X_c(e^{j\Omega})$ is given by,

$$\text{Max}\left\{X_c(e^{j\Omega})\right\} = T\left[\text{Max}\left\{X_d(e^{j\Omega})\right\}\right]$$

$$= (0.5 \times 10^{-3})(1)$$

$$= 0.5 \times 10^{-3}$$

Thus,

$$\boxed{\text{Max}\left\{X_c\left(e^{j\Omega}\right)\right\} = 0.5 \times 10^{-3}}$$

(c)

From the given equation $x_d[n] = x_c\left(n(0.5 \times 10^{-3})\right)$ the following equation can be deduced,

$$x_d[n] = x_c(nT)$$

Where $T = 0.5$ ms

The effect of that in the frequency domain is that,

$$X_c(e^{j\omega}) = 0 \text{ for } \frac{3\pi}{4T} \leq |\omega| \leq \frac{\pi}{T}$$

Then, the range of ω for which $X_c(e^{j\omega}) = 0$ is given by,

$$\frac{3\pi}{4T} \leq |\omega| \leq \frac{\pi}{T}$$

$$\frac{3\pi}{4(0.5 \times 10^{-3})} \leq |\omega| \leq \frac{\pi}{0.5 \times 10^{-3}}$$

$$1500\pi \leq |\omega| \leq 2000\pi$$

From the givens, $X_c(j\omega) = 0$ for $|\omega| \geq 1500\pi$.
Thus,

$$\boxed{|\omega| \geq 1500\pi}$$

(d)

From the given equation $X_d(e^{j\omega}) = X_d(e^{j(\omega-\pi)})$, the value of the discrete frequency Ω equals π.

Then, the value of the continuous frequency ω is given by,

$$\omega = \frac{\Omega}{T}$$

$$= \frac{\pi}{0.5 \times 10^{-3}}$$

$$= 2000\pi \text{ rad/s}$$

Thus,

$$\boxed{X_c(j\omega) = X_c(j(\omega - 2000\pi)) \text{ for } 0 \leq |\omega| \leq 2000\pi}$$