

Signal and System Final Exam

201/01/08

班級:

姓名:

學號:

※請將詳細計算過程寫在答案卷上，並於考試結束後繳回試卷與答案紙
共八題，滿分 100 分

1. Consider a continuous-time ideal lowpass filter S whose frequency response is

$$H(j\omega) = \begin{cases} 1, & |\omega| \leq 100 \\ 0, & |\omega| > 100 \end{cases}$$

When the input to this filter is a signal $x(t)$ with fundamental period $T = 4\pi/3$ and Fourier series coefficients a_k , it is found that

$$x(t) \xrightarrow{S} y(t) = x(t)$$

For what values of k is it guaranteed that $a_k=0$?

2. Use the Fourier transform analysis equation to calculate the Fourier transforms of :

(a) $e^{-2(t-1)}u(t-1)$

(b) $\delta(t+1) - \delta(t-1)$

And use the Fourier transform synthesis equation to determine the inverse Fourier transforms of:

(c) $X_1(j\omega) = \pi\delta(\omega - 2\pi) - \pi\delta(\omega + 2\pi)$

(d) $X_2(j\omega) = \begin{cases} 2, & 0 \leq \omega \leq 2 \\ -2, & -2 \leq \omega < 0 \\ 0, & |\omega| > 2 \end{cases}$

3. Use the sinc function to describe the following signal:

$$x(t) = \cos(\pi t)$$

4. (a) Determine the Fourier transform of the following signal:

$$x(t) = t \left(\frac{\sin t}{\pi t} \right)^2$$

- (b) Use Parseval's relation and the result of the previous part to determine the numerical value of

$$A = \int_{-\infty}^{+\infty} t^2 \left(\frac{\sin t}{\pi t} \right)^4 dt$$

5. Consider a causal LTI system with frequency response

$$H(j\omega) = \frac{1}{j\omega + 4}$$

For a particular input $x(t)$ this system is observed to produce the output

$$y(t) = e^{-3t}u(t) - e^{-4t}u(t)$$

Determine $x(t)$.

6. Determine the Fourier transform for $-\pi \leq \omega < \pi$ in the case of each of the following periodic signals:

(a) $\cos\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$

(b) $5 + \sin\left(\frac{\pi}{6}n + \frac{\pi}{8}\right)$

7. The following four facts are given about a real signal $x[n]$ with Fourier transform $X(e^{j\omega})$:

1. $x[n] = 0$ for $n > 0$

2. $x[0] > 0$

3. $\text{Im}\{X(e^{j\omega})\} = \sin 2\omega - \sin 3\omega$.

4. $\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 5$

Determine $x[n]$.

8. Determine the Nyquist rate corresponding to each of the following signals:

1. $x(t) = \cos(2\pi t) + \sin(3\pi t)$

2. $x(t) = \left(\frac{\sin(377\pi t/2)}{\pi t}\right)^2$