Yuntech EE - Signals and Systems Final Examination

Name: ____________________ Student ID: ____________________ 95/01/9

1. (20%) Consider the causal continuous-time system described by a first order differential equation

$$\frac{dy(t)}{dt} + ay(t) = bx(t),$$

which can be rewritten as

$$y(t) = -\frac{1}{a}\frac{dy(t)}{dt} + \frac{b}{a}x(t).$$

Please draw two block diagrams: (a) using the basic operations: addition, multiplication by a coefficient, and differentiation. (b) using the equation

$$y(t) = \int_{-\infty}^{t} [bx(\tau) - ay(\tau)]d\tau$$

and the basic operations: addition, multiplication by a coefficient, and integration.

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2. (10%) For the continuous-time periodic signal

$$x(t) = 2 + 3\cos(\frac{2\pi}{3})t + 4\sin(\frac{5\pi}{3}t),$$

determine the fundamental frequency w_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}.$$

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3. (10%) If $x(t)$ is a real and periodic signal, then $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}$. Please derive the alternative form of Fourier series shown below:

$$x(t) = a_0 + \sum_{k=1}^{\infty} 2a_k \cos(kw_0 t).$$

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4. (10%) For an input $x(t) = e^{st}$, we can determine the output through the use of the convolution integral, i.e.,

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = H(s)e^{st}.$$

Please show that

$$H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau.$$

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5. (10%) In Problem 4, if $y(t) = x(t-2) + x(t-3)$, that is, the impulse response of system is $h(t) = \delta(t-2) + \delta(t-3)$, please determine the function $H(s)$.

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6. (10%) (a) Consider an LTI system with input and output related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2) d\tau.$$

What is the impulse response $h(t)$ for this system?
(b) Determine the response of the system when the input is $x(t) = u(t+1) - u(t-2)$.

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7. (10%) Determine the fundamental period of the discrete-time signal $x[n] = e^{j(2\pi/5)n} - e^{j(3\pi/4)n}$.

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8. (10%) Please determine the signal $y(t)$ as the convolution of the following two signals: $x(t) = e^{3t}u(-t)$ and $h(t) = u(t-2)$.

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9. (10%) Let $x(t) = e^{-a|t|}$, $a > 0$. Please determine the Fourier transform of the signal $x(t)$.

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10. (12%) Consider that a discrete-time signal $x[n]$ is periodic with period L, the fundamental frequency w_0 becomes __________. If a_k is the Fourier series coefficients, the discrete-time Fourier series representation of $x[n]$ can be expressed as

$$x[n] = \underline{\hspace{6cm}},$$

where a_k can be determined from $x[n]$ by the use of the equation

$$a_k = \underline{\hspace{6cm}}.$$

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11. (10%) Consider the signal

$$x[n] = \sin w_0 n = \frac{1}{2j}(e^{jw_0 n} - e^{-jw_0 n}), \text{ with } w_0 = \frac{2\pi}{5}.$$

Please determine the Fourier transform of $x[n]$ and depict $X(e^{jw})$ within one period.

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12. (10%) Use Tables 5.1 and 5.2 to help determine $x[n]$ when its Fourier transform is

$$X(e^{jw}) = \frac{1}{1 - e^{-jw}}\left(\frac{\sin \frac{3}{2}w}{\sin \frac{w}{2}}\right) + 5\pi\delta(w), \quad -\pi < w \leq \pi.$$

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13. (8%) Let $x[n]$ be a real and odd periodic signal with period $N = 7$ and Fourier coefficients a_k. Given that $a_{15} = j$, $a_{16} = 2j$, $a_{17} = 3j$, determine the values of a_0, a_{-1}, a_{-2}, and a_{-3}.

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14. (10%) The definition of a sinc function is $\text{sinc}(\theta) = \frac{\sin \pi\theta}{\pi\theta}$. Please rewrite the following signal in terms of the sinc functions :

$$\frac{\sin 2Wt}{\pi t} = \underline{\hspace{5cm}}.$$

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15. (10%) For a discrete-time signal $x[n] = a^{|n|}$, please determine its discrete-time Fourier transform $X(e^{jw})$.
