Yuntech EE - *Signals and Systems* Final Exam

Name: _____ Student ID: _____ 97/01/16

1. (Chapter 3, 10%) Let $x[n]$ be a ***real and odd*** periodic signal with period $N = 9$ and Fourier coefficients a_k. Given that $a_{12} = -j$, $a_{11} = -2j$, $a_{10} = -3j$, determine the values of a_0,a_{-1}, a_{-2}, and a_{-3}.

2. (Chapter 3, 20%) Consider an LTI system with impulse response $h[n] = \alpha^n u[n], -1 < \alpha < 1$, and with the input $x[n] = \sin(\frac{2\pi n}{N}) = \frac{1}{2j}\left\{\frac{1}{2}e^{j(2\pi/N)n} - \frac{1}{2}e^{-j(2\pi/N)n}\right\}$. The frequency response of the system (Fourier transform of $h[n]$) is $H(e^{jw}) = \sum_{n=0}^{\infty}\alpha^n e^{-jwn} = \sum_{n=0}^{\infty}(\alpha e^{-jw})^n$. Please determine the system output $y[n] = h[n] * x[n]$.

3. (Chapter 4, 10%) The definition of a **sinc** function is $\text{sinc}(\theta) = \frac{\sin\pi\theta}{\pi\theta}$. Please rewrite the following signal $\frac{\sin(5Wt)}{3\pi t}$ in terms of the sinc functions.

4. (Chapter 4, 10%) Determine the Fourier transform of the periodic signal $1 + 2\sin(6\pi t) + 3\cos(2\pi t)$.

5. (Chapter 4, 20%) Consider the Fourier transform pair $e^{-|t|} \leftrightarrow \frac{2}{1+\omega^2}$.
   (a) Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$.
   (b) Use the result from part (a), along with the ***duality property***, to determine the Fourier transform of $\frac{4t}{(1+t^2)^2}$.

6. (Chapter 5, 15%) Use Tables 5.1 and 5.2 to help determine $x[n]$ when its Fourier transform is
$$X(e^{jw}) = \frac{1}{1-e^{-jw}}\left(\frac{\sin\frac{3}{2}w}{\sin\frac{w}{2}}\right) + 5\pi\delta(w), \quad -\pi < w \leq \pi.$$

7. (Chapter 5, 15%) Use Tables 5.1 and 5.2 to determine (a) the Fourier transform of the signal $x[n] = n(\frac{3}{5})^n u[n]$ and (b) the value of $X(e^{j0})$.

Good Luck and Have Happy Winter Vacation!