Signals and Systems Quiz #6 (Sec. 5.1–5.7)

Name: ____________________ ID No.: ____________________ 2004/12/30

1. (10%) For an aperiodic discrete-time signal $x[n]$, please give its Fourier transform pair as follows:

$$x[n] = \underline{\hspace{5cm}}, \qquad X(e^{jw}) = \underline{\hspace{5cm}}.$$

2. (10%) The Fourier transform $X(e^{jw})$ of the discrete-time signal $x[n]$ will often be referred to as the ________________ of $x[n]$, because it provides us with the information on how $x[n]$ is composed of complex exponentials at different frequencies.

3. (10%) Signals at frequencies near $w = 0$ and $w = 2\pi$ or any other even-multiple of π are all appropriately thought of as ____________________ signals. On the other hand, signals at frequencies near odd multiple of π are can be thought of as ____________________ signals.

4. (20%) Let $x[n] = a^{|n|}$, $|a| < 1$. Please determine its Fourier transform $X(e^{jw})$.

5. (10%) (a) The Fourier transform of the signal $x[n] = e^{jw_0 n}$ is the impulse train

$$X(e^{jw}) = \sum_{l=-\infty}^{\infty} 2\pi\delta(\underline{\hspace{4cm}}).$$

(10%) (b) Consider a periodic sequence $x[n]$ with period N and with the Fourier series representation

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}.$$

The Fourier transform is

$$X(e^{jw}) = \sum_{k=-\infty}^{\infty} \underline{\hspace{6cm}}$$

6. (10%) For the discrete-time periodic impulse train

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - kN],$$

the Fourier series coefficients is $a_k = \frac{1}{N}$. The Fourier transform consists of a train of impulses in the frequency domain, with the area of the impulses is ________________.

7. (20%) Consider the signal $x[n] = \sin w_0 n = \frac{1}{2j}(e^{jw_0 n} - e^{-jw_0 n})$, with $w_0 = \frac{2\pi}{5}$. Please determine the Fourier transform of $x[n]$ and depict $X(e^{jw})$ within one period.

8. (10%) The discrete-time Fourier transform of an aperiodic signal is always periodic in w with period __________.

9. (10%) The convolution and multiplication properties:

$$y[n] = x[n] * h[n] \xleftarrow{\mathcal{F}}\!\!\rightarrow Y(e^{jw}) = \underline{\hspace{6cm}}$$

$$y[n] = x_1[n]x_2[n] \xleftarrow{\mathcal{F}}\!\!\rightarrow Y(e^{jw}) = \underline{\frac{1}{2\pi}\hspace{6cm}}.$$

10. (20%) Suppose that the frequency response of an ideal lowpass filter is denoted as $H_{\text{lp}}(e^{jw})$ with cutoff frequency w_c. Please explain how an ideal highpass filter with cutoff frequency $\pi - w_c$ can be represented by an lowpass filter shifted by one-half period, i.e.,

$$H_{\text{hp}}(e^{jw}) = H_{\text{lp}}(e^{j(w-\pi)}).$$