Chapter 1

Signals and Systems – *basic concepts*

1.0 Introduction

- 本課程在所有工程科技領域都有基本而重要的應用，且歷久彌新
- Communications 通訊
- Audio and Speech processing 語音訊號處理
- Image and Video Processing 影像與視訊處理
- Acoustics 聲波學
- Circuit Design 電路設計
- Seismology 地震學
- Biomedical Engineering 生醫工程
- Bioinformatics 生物資訊
- Energy Generation and Distribution System 能量產生與配送系統
- Chemical Process Control 化學程序控制
- Aeronautics 航空學
- Astronautics 太空航行學
- …

Summary

- This course is one of several ***fundamental*** required courses for Electrical Engineering. It covers analytic (***mathematic***) background for modeling and analyzing real-world signals and systems.

- Examples of signals include those involving electricity, ***audio, images, video, radar signals, and seismic signals***.

- Systems ***store, manipulate, or transmit*** signals by **physical processes**. Examples include electric circuits and systems, communication systems, control systems, and signal processors.

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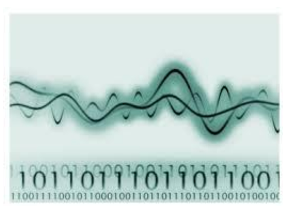
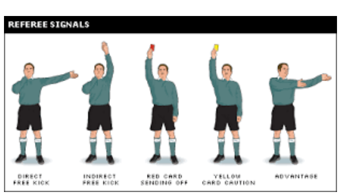
What is a signal?





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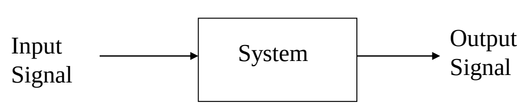
1.1 Signals

- Signals may describe a wide variety of physical phenomena. It can represent mathematically as ***functions*** of one or more independent variables: $f(..x,y,z..)$, $x(t)$

- For example:
  - Time: $x(t)$
  - Frequency: $X(f)$
  - Temperature ^oF, ^oC
  - Pressure
  - And many others…



Examples of Systems:
- Figure 1.1 A simple RC circuit (Page 2)
- Figure 1.2 An automobile

Examples of Signals:
- Figure 1.3 Example of recorded speech.
- Figure 1.4 A Monochromatic Picture
- Figure 1.5 Typical vertical wind profile
- Figure 1.6 Weekly Dow-Jones Stock Market

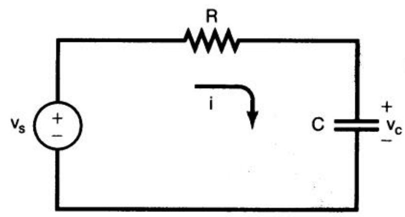


圖 1.1 具有電源 v_s 和電容電壓 v_c 的簡單 *RC* 電路

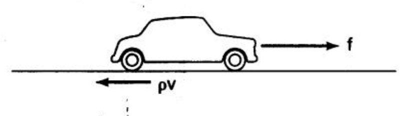


圖 1.2 汽車引擎產生的應用力 f 和與汽車速度 v 成正比的摩擦力 ρv 的響應

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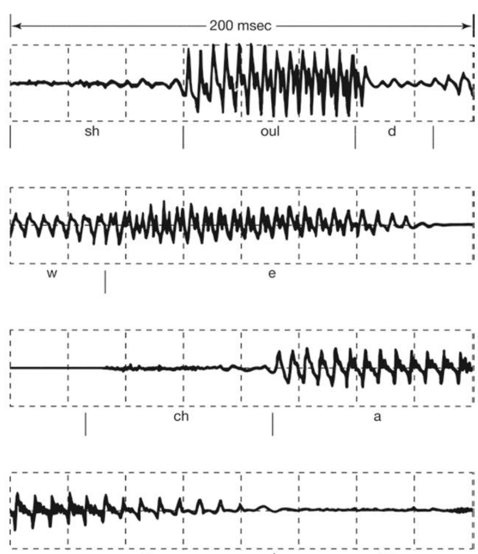

圖 1.3 某個語音信號的波形。圖的第一行對應到 "should" 這個字，第二行的字是 "we"，最後兩行的字是 "chase"。（在每個字中，我們也大約標示出每個連續聲音的開始和結束。）

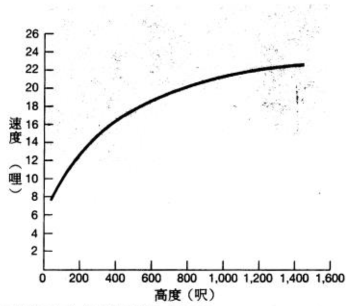

圖 1.5 典型年平均沿垂直方向風速分佈圖（摘取自 Crawford and Hudson, National Severe Storms Laboratory Report, ESSA ERLTM-NSSL 48, August 1970）

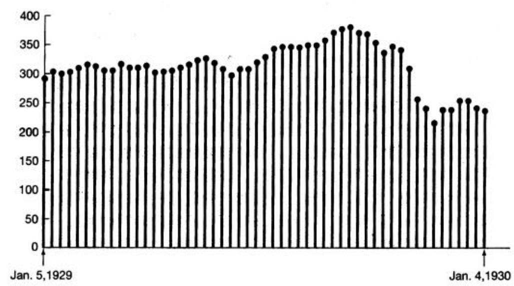

圖 1.6 離散時間信號的例子：從 1929 年 1 月 5 日至 1930 年 1 月 4 日，每週股票市場道瓊指數的變化

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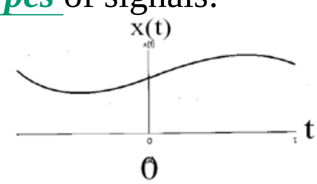
We will be considering ***two basic types*** of signals:

- ***Continuous-time*** **signals** $\boldsymbol{x(t)}$



We enclose the independent variable in parentheses (·)

- ***Discrete-time*** **signals** $\boldsymbol{x[n]}$

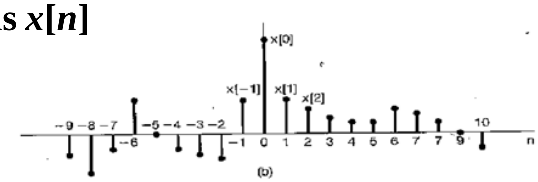


Figure 1.7 Graphical representations of (a) continuous-time and (b) discrete-time signals.

We use brackets [·] to enclose the independent variable.

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1.1.2 Signal Energy and Power

- The ***energy*** (E) and ***power*** (P) of ***continuous-time*** signal $x(t)$ over the time interval $t_1 \leq t \leq t_2$:

$$E = \int_{t_1}^{t_2} |x(t)|^2 dt, \qquad P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

- The ***energy*** (E) and ***power*** (P) of ***discrete-time*** signal $x[n]$ over the time interval $n_1 \leq n \leq n_2$:

$$E = \sum_{n=n_1}^{n_2} |x[n]|^2 \,, P = \frac{1}{(n_2 - n_1) + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

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- In many systems, we are interested in examining power and energy in signals ***over an infinite time interval***, *i.e.*, for $-\infty \leq t \leq +\infty$ or $-\infty \leq n \leq +\infty$, then write as follow…

- Continuous-time signal $x(t)$

$$E = \lim_{T\to\infty} \int_{-T}^{+T} |x(t)|^2 \, dt, \qquad P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 \, dt$$

- Discrete-time signal $x[n]$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{+N} |x[n]|^2 \,, P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$

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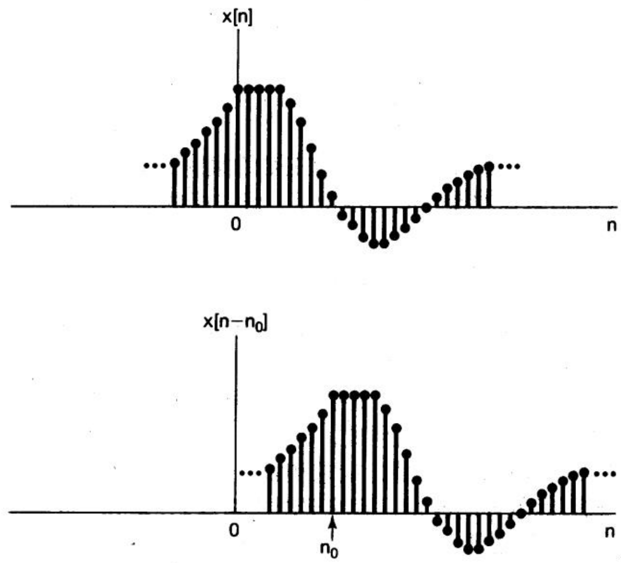

圖 1.8 有時間移位的離散時間信號。圖中 $n_0 > 0$，故 $x[n-n_0]$ 是 $x[n]$ 的延遲（即 $x[n]$ 的每點稍後出現在 $x[n-n_0]$）.

1.2 Transformations of Independent Variable

A central concept in signal and system analysis is that of the ***transformation*** of a signal.

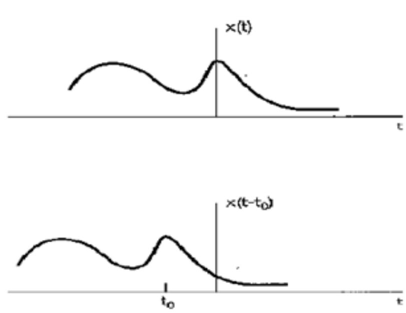

Figure 1.9 Continuous-time signals related by a time shift. In this figure $t_0 < 0$, so that $x(t-t_0)$ is an advanced version of $x(t)$ (i.e., each point in $x(t)$ occurs at an earlier time in $x(t-t_0)$).

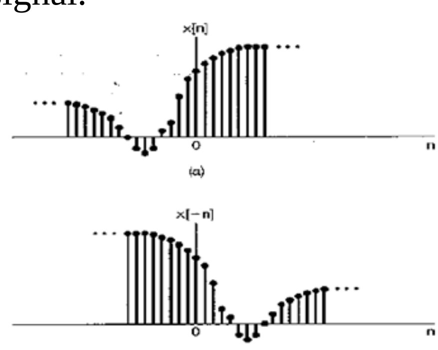

Figure 1.10 (a) A discrete-time signal $x[n]$; (b) its reflection $x[-n]$ about $n = 0$.

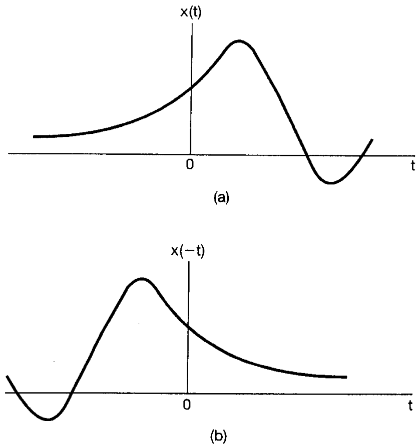

Figure 1.11 (a) A continuous-time signal $x(t)$; (b) its reflection $x(-t)$ about $t = 0$.

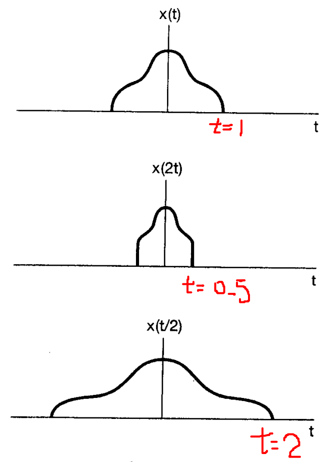

Figure 1.12 Continuous-time signals related by time scaling.

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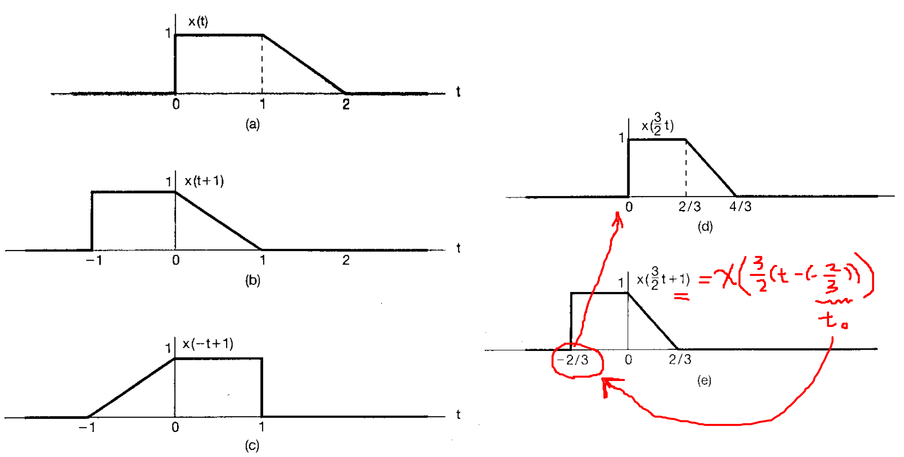

Figure 1.13 (a) The continuous-time signal $x(t)$ used in Examples 1.1–1.3 to illustrate transformations of the independent variable; (b) the time-shifted signal $x(t + 1)$; (c) the signal $x(-t + 1)$ obtained by a time shift and a time reversal; (d) the time-scaled signal $x(\frac{3}{2}t)$; and (e) the signal $x(\frac{3}{2}t + 1)$ obtained by time-shifting and scaling.

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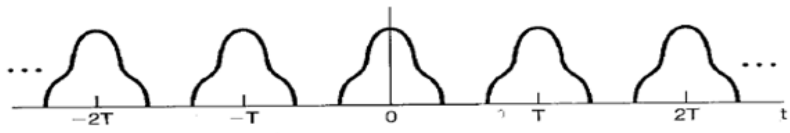
1.2.2 Periodic Signals

- Continuous-time → $x(t)=x(t+\boldsymbol{T})$, for all values of t
  It's unchanged by a ***time shift*** of T.
  For example, $x(t)=x(t+\boldsymbol{mT})$, m is an integer.



- Discrete-time → $x[n]=x[n+\boldsymbol{N}]$, for all values of n
  It's unchanged by a time shift of N.

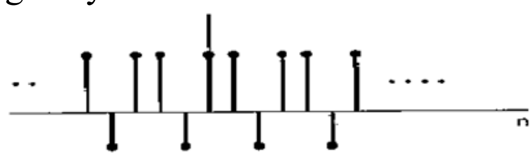


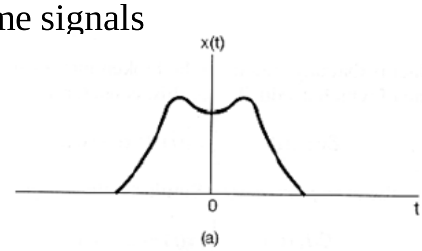
Fig. 1.15

1.2.3 Even and Odd signals

Even signals: 偶函數信號

$x(-t) = x(t)$, for continuous-time signals

$x[-n] = x[n]$, for discrete-time signals



Odd signals: 奇函數信號

$x(-t) = -x(t)$, for continuous-time signals

$x[-n] = -x[n]$, for discrete-time signals

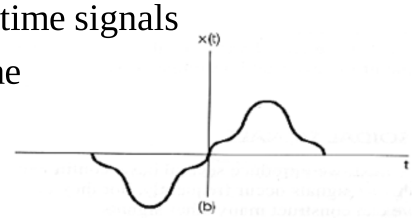
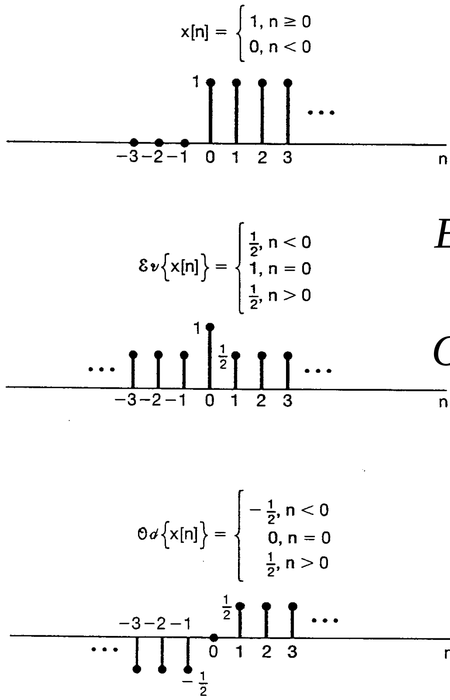


Fig. 1.17



$$Ev\{x[n]\} = \frac{1}{2}\{x[n] + x[-n]\}$$

$$Od\{x[n]\} = \frac{1}{2}\{x[n] - x[-n]\}$$

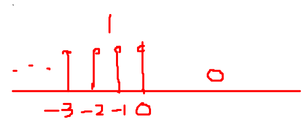


Figure 1.18 Example of the even-odd decomposition of a discrete-time signal.

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1.3.1 Exponential and sinusoidal signals

- Continuous-time signal

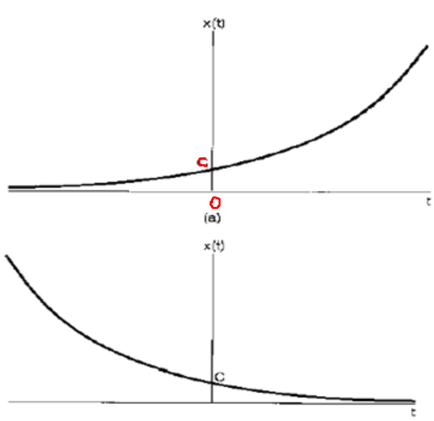

Fig. 1.19

$x(t) = Ce^{at}$

(a) $a>0$;

(b) $a<0$;

(c) $a=0$, $x(t)=C$ i.e. a constant

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Periodic Complex Exponential and Sinusoidal Signals

Consider $x(t) = e^{jw_0 t}$

for a periodic signal with period *T*,

then $e^{jw_0 t} = e^{jw_0 (t+T)}$

Or since $e^{jw_0 (t+T)} = e^{jw_0 t} e^{jw_0 T}$

We must have $e^{jw_0 T} = 1$

The fundamental period $T_0 = \frac{2\pi}{|\omega_0|}$

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Euler's Relation $e^{j\theta} = \cos\theta + j\sin\theta$

$$e^{jw_0 T} = \cos w_0 T + j\sin w_0 T = 1$$

$$\cos w_0 T = 1 \text{ and } \sin w_0 T = 0$$

$$w_0 T = 2n\pi, \; n \text{ is an integer}$$

For the nonzero and smallest integer, *n*=+1 or -1

$$|w_0| T_0 = 2\pi, \;\; \Rightarrow T_0 = \frac{2\pi}{|w_0|}$$

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A signal closely related to the periodic complex exponential is the sinusoidal signal

$$x(t) = A\cos(w_{\mathrm{o}}t + \emptyset)$$

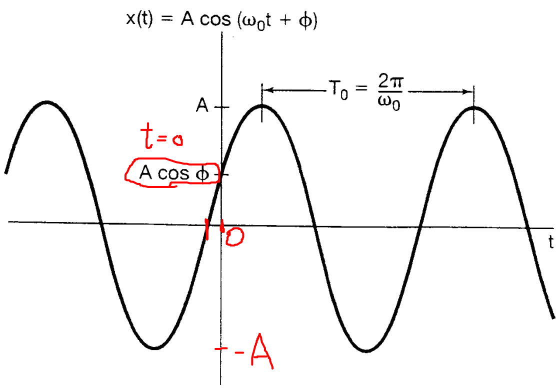

Figure 1.20 Continuous-time sinusoidal signal.

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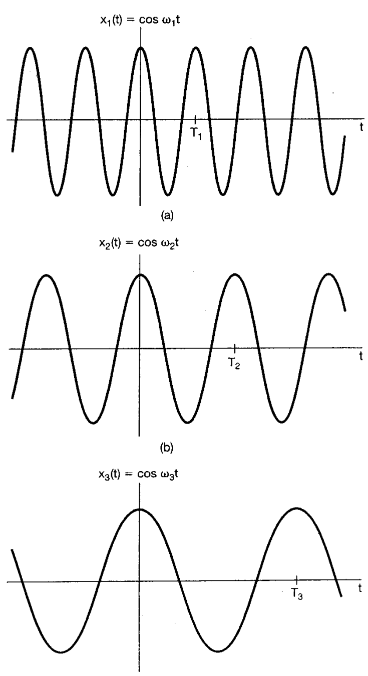

Figure 1.21 Relationship between the fundamental frequency and period for continuous-time sinusoidal signals; here, $\omega_1 > \omega_2 > \omega_3$, which implies that $T_1 < T_2 < T_3$.

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Example1.5

General Complex Exponential Signals

Consider a complex exponential Ce^{at}, where ***C*** is expressed in polar form and ***a*** in rectangular form. That is,

$$C = |C|e^{j\theta} \quad \text{and} \quad a = r + jw_0$$

Then

$$Ce^{at} = |C|e^{j\theta}e^{(r+jw_0)t} = |C|e^{rt}e^{j(w_o t+\theta)}$$
$$= |\mathrm{C}|e^{rt}\cos(w_0 t + \theta) + j|C|e^{rt}\sin(w_0 t + \theta)$$

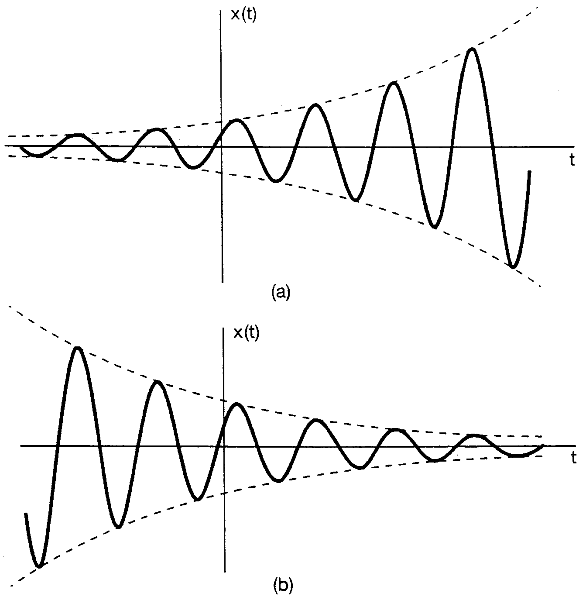

Figure 1.23 (a) Growing sinusoidal signal $x(t) = Ce^{rt}\cos(\omega_0 t + \theta)$, $r > 0$; (b) decaying sinusoid $x(t) = Ce^{rt}\cos(\omega_0 t + \theta)$, $r < 0$.

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1.3.2 Exponential and sinusoidal signal

- Discrete-time signal
- $x[n] = C\alpha^n$, **C and α are real**

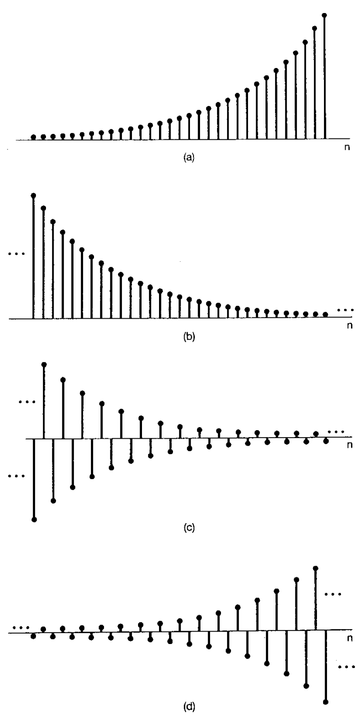

Figure 1.24 The real exponential signal $x[n] = C\alpha^n$: (a) $\alpha > 1$; (b) $0 < \alpha < 1$; (c) $-1 < \alpha < 0$; (d) $\alpha < -1$.

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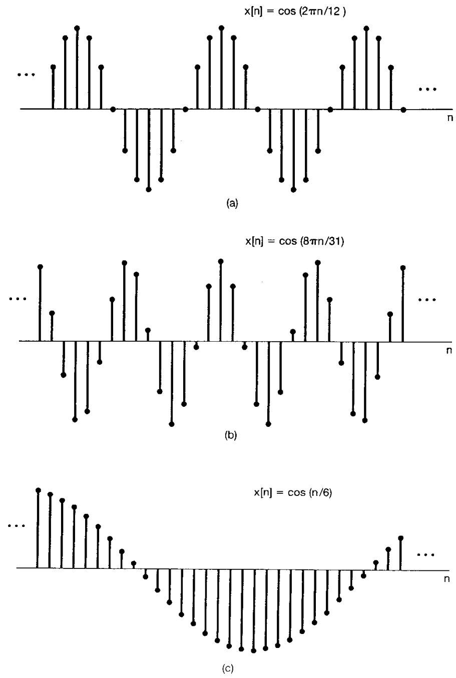

Figure 1.25 Discrete-time sinusoidal signals.

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General Complex Exponentioal Signals: C and α are complex

- $C = |C|e^{j\theta}$
- $\alpha = |\alpha|\ e^{j\omega_0}$
- Then, $C\alpha^n = |C||\alpha|^n e^{j(\omega_0 n+\theta)}$

$= |C||\alpha|^n \cos(\omega_0 n + \theta) + j|C||\alpha|^n \sin(\omega_0 n + \theta)$

- For $|\alpha|$ =1, the real and imaginary parts of a complex exponential sequence are sinusoidal.

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C and α are complex, p. 25, Eq. (1-50)

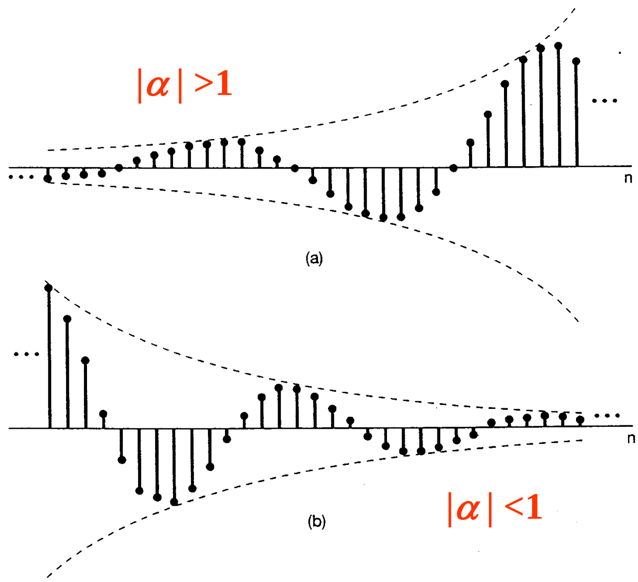

Figure 1.26 (a) Growing discrete-time sinusoidal signals; (b) decaying discrete-time sinusoid.

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1.3.3 Periodicity Properties of Discrete-Time Complex Exponentials

$$e^{j(w_0+2\pi)n} = e^{j2\pi n}e^{jw_0 n} = e^{jw_0 n}.$$

$$e^{j\pi n} = \left(e^{j\pi}\right)^n = (-1)^n$$

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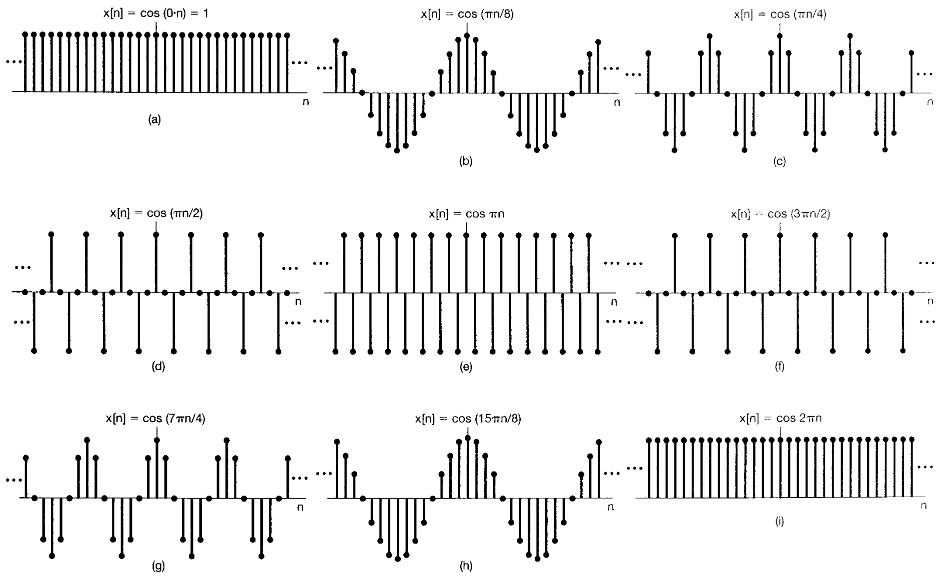

Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

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TABLE 1.1 Comparison of the signals $e^{j\omega_0 t}$ and $e^{j\omega_0 n}$.

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct values of ω_0	Identical signals for values of ω_0 separated by multiples of 2π
Periodic for any choice of ω_0	Periodic only if $\omega_0 = 2\pi m/N$ for some integers $N > 0$ and m.
Fundamental frequency ω_0	Fundamental frequency* ω_0/m
Fundamental period $\omega_0 = 0$: undefined $\omega_0 \neq 0$: $\frac{2\pi}{\omega_0}$	Fundamental period* $\omega_0 = 0$: undefined $\omega_0 \neq 0$: $m\left(\frac{2\pi}{\omega_0}\right)$

*Assumes that m and N do not have any factors in common.

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Example 1.6

- Determine the fundamental period of the discrete-time signal $x[n]=e^{j(2\pi/3)n}+e^{j(3\pi/4)n}$.

- The first exponential on the right-hand side has a fundamental period of 3. ($3\times 2\pi/3=2\pi$)
- For the second term, the fundamental period is 8. ($8\times 3\pi/4=6\pi$, the smallest multiple of 2π)
- The smallest increment of n that simultaneously satisfies the two terms is 24.

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1.4 The unit impulse and unit step function

1.4.1 The Discrete-Time Unit impulse and unit step sequences

- Unit pulse:

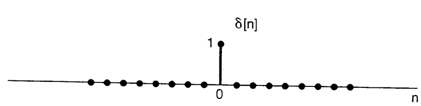

Figure 1.28 Discrete-time unit impulse (sample).

$$\delta[n]=\begin{cases}0, & n\neq 0\\ 1, & n=0\end{cases}$$

- Unit step:

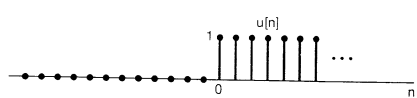

Figure 1.29 Discrete-time unit step sequence.

$$u[n]=\begin{cases}0, & n<0\\ 1, & n\geq 0\end{cases}$$

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$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] = \sum_{k=0}^{\infty} \delta[n-k]$$

$x[n]\delta[n] = x[0]\delta[n], \delta[n]$ is non zero only for $n = 0$.

Similarly, $x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$

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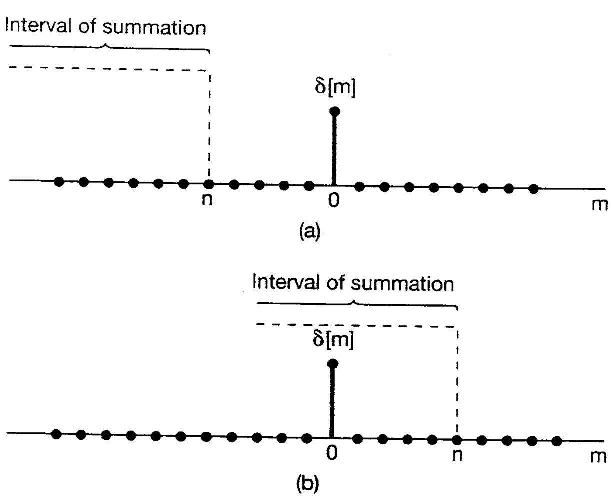

Figure 1.30 Running sum of eq. (1.66): (a) $n < 0$; (b) $n > 0$.

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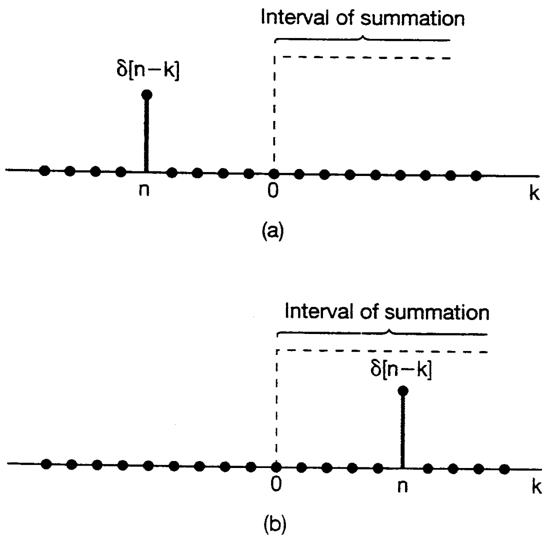

Figure 1.31 Relationship given in eq. (1.67): (a) $n < 0$; (b) $n > 0$.

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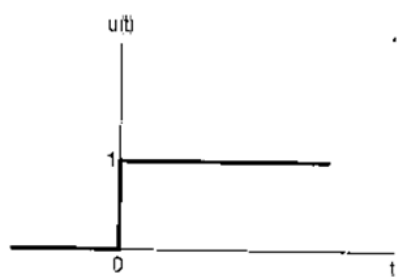
1.4.2 The Continuous-time unit impulse and unit step sequences

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = \int_{0}^{\infty} \delta(t-\sigma)d\sigma .$$

$$\delta(t) = \frac{du(t)}{dt} .$$



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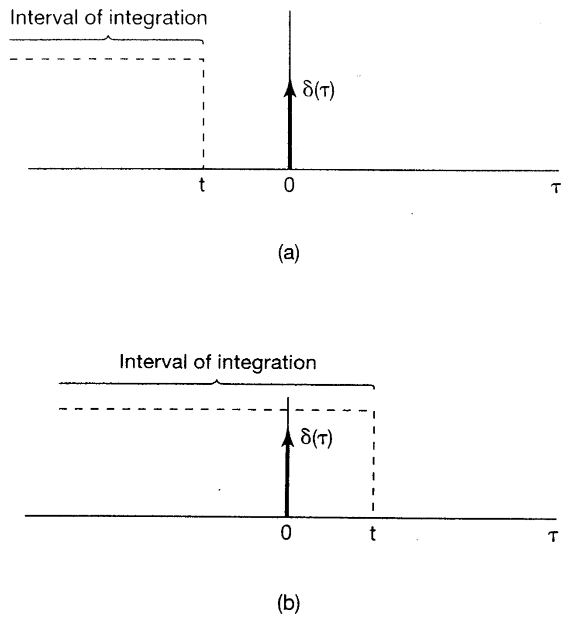

Figure 1.37 Running integral given in eq. (1.71): (a) $t < 0$; (b) $t > 0$.

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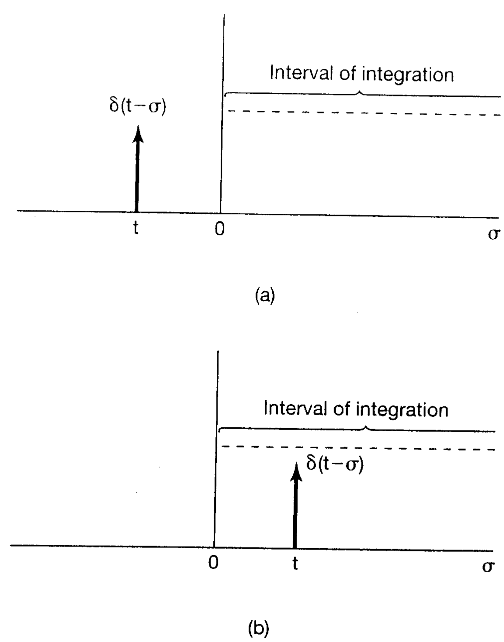

Figure 1.38 Relationship given in eq. (1.75): (a) $t < 0$; (b) $t > 0$.

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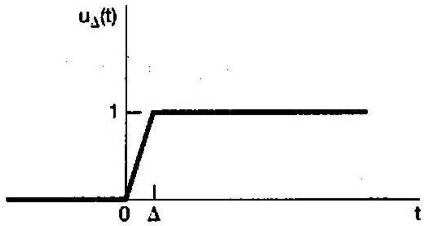

圖 1.33 單位步級 $u_\Delta(t)$ 的連續近似

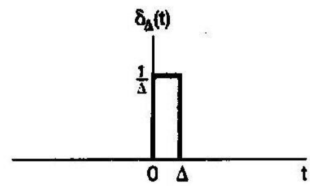

圖 1.34 $u_\Delta(t)$ 的導數

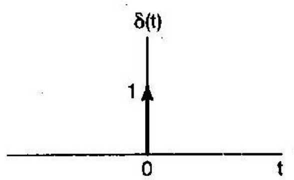

圖 1.35 連續時間單位脈衝

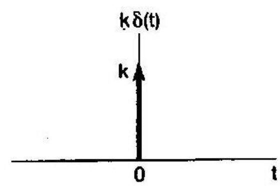

圖 1.36 脈衝強度 k 的脈衝

1.5 Continuous-time and discrete-time system representation

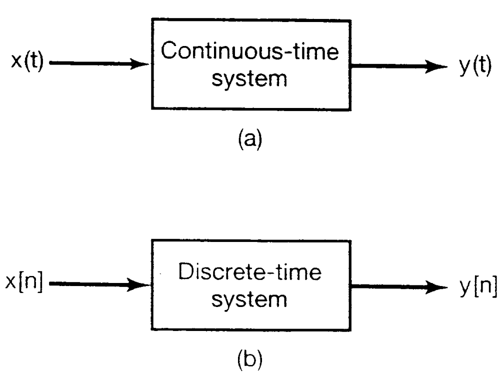

Figure 1.41 (a) Continuous-time system; (b) discrete-time system.

1.5.1 Simple Examples of Systems

- Example 8 – the RC circuit depicted in Fig. 1.1
- Example 9 – Fig. 1.2
- Example 10 – A simple model of the balance in a bank account from month to month
- Example 11 – a simple digital simulation of the differentiation equation

1.5.2 Interconnections of systems

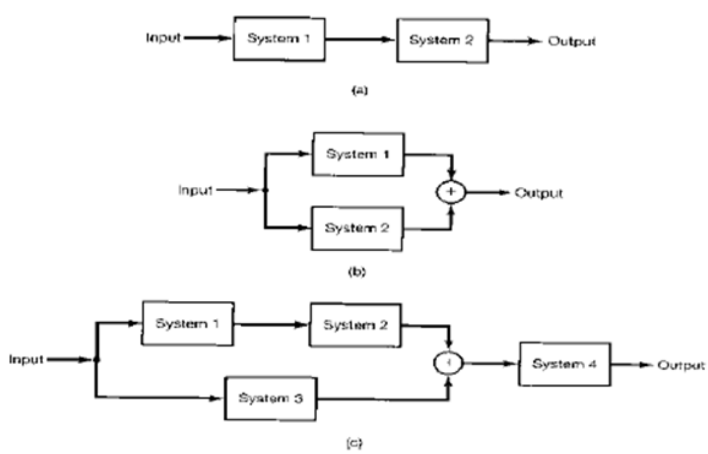

Figure 1.42 Interconnection of two systems: (a)series(cascade); (b) parallel; (c)series-parallel.

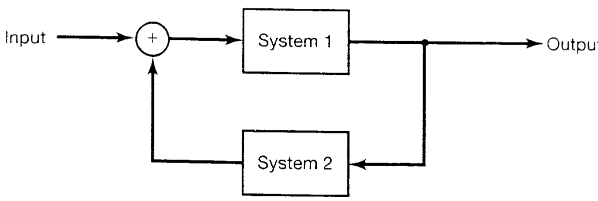

Figure 1.43 Feedback interconnection.

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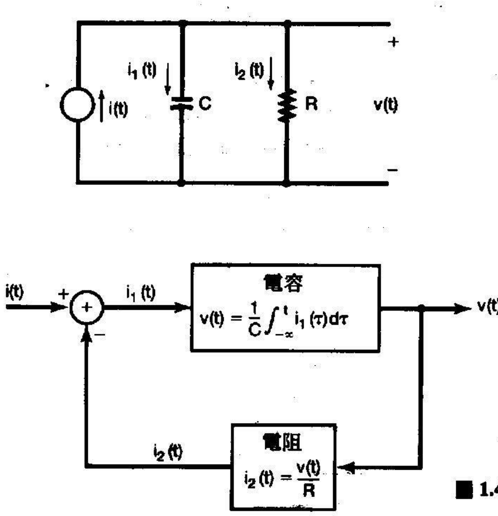

圖 1.44 (a)簡單的電子電路; (b)將(a)的電路畫成兩個電路元件反饋互聯的方塊圖

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1.6 Basic system properties

- 1.6.1 Systems with and without memory
  - ***Memoryless*** system: its output for each value of the independent variable at a given time is dependent on the input at only that same time.
  - For example: $y[n]=(2x[n]- x^2[n])^2$
  - An example of a discrete-time system with ***memory*** is an accumulator of summer: $y[n] = \sum_{k=-\infty}^{n} x[k].$
  - Second example: Delay: $y[n]=x[n-1]$

The concept of memory in a system corresponds to the presence of a mechanism in the system that retains or stores information about input values at times other than the current time.

- Continuous-time system with memory:
  - Capacitor: input is current and output is the voltage (C is the capacitance)

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau$$

- Accumulator: add the current input value to the preceding value of the running sum.

$$y[n] = \sum_{k=-\infty}^{n-1} x[k] + x[n] = y[n-1] + x[n]$$

1.6.2 Invertibility and Inverse system

- A system is said to be ***invertible*** if distinct inputs lead to distinct outputs. For example,

  $y(t)=2x(t).$

  For which the inverse system is

  $w(t)={}^1/_2 y(t)$

- Examples of noninvertible systems are

  $y[n]=0$, produces zero output sequence for any input sequence.

  $y(t)=x^2(t)$, can't determine the sign of the input from knowledge of the output

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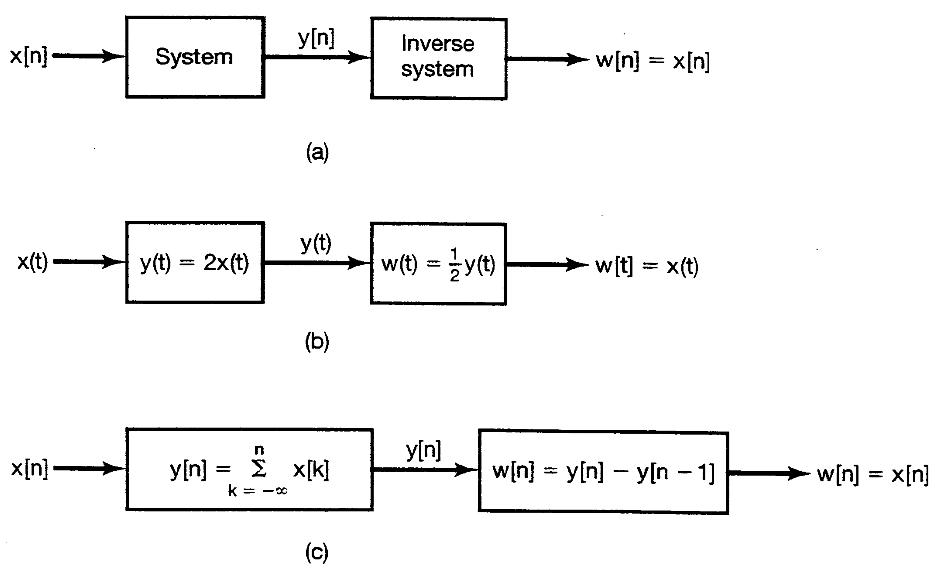

Figure 1.45 Concept of an inverse system for: (a) a general invertible system; (b) the invertible system described by eq. (1.97); (c) the invertible system defined in eq. (1.92).

- **1.6.3 Causality**
  - A system is ***causal*** if the output at any time depends only on values of the input at the ***present time and in the past.***
  - All **memoryless** systems are causal since the output responds only to the current value of the input.

Causal example: $y[n]=x[-n]$, $n>0$

Non-causal example: $y[-4]=x[4]$

$y(t)=x(t)\cos(t+1)$ → causal or noncausal?

1.6.4 Stability

- **BIBO** property: Bounded input leads to bounded output
- $y(t)=tx(t)$ → stable? ***No***
- If $x(t)=1$ (bounded input), then $y(t)=t$, which is unbounded.

- $y(t)=\exp[x(t)]$ → stable? ***Yes***
- If $|x(t)|< B$, $-B<x(t)<B$ (bounded input), then $\exp[-B]<\exp[x(t)]=y(t)<\exp[B]$, output is bounded.

1.6.5 Time Invariance

Conceptually, a system is ***time invariant*** if the behavior and characteristics of the system are ***fixed over time***. For example, the RC circuit of figure shown below is time invariant if the resistance and capacitance values R and C are constant over time.

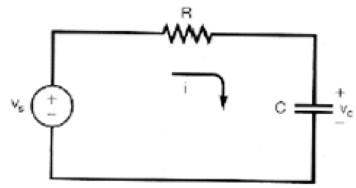

Figure 1.1 A simple *RC* circuit with source voltage v_s and capacitor voltage v_c.

Example of time invariance:

$x[n] \rightarrow y[n]$ after a time shifting of input signal $x[n-1]$

$x[n-1] \rightarrow y[n-1]$ identical time shit in the output signal $y[n-1]$

Examples 1.14 & 1.15

Example 1.16

Consider the system $y(t)=x(2t)$, ***time variant***

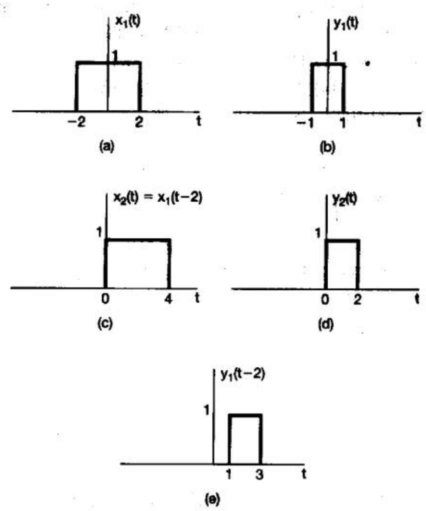

圖 1.47 (a)對例題 1.16系統的輸入 $x_1(t)$; (b) $x_1(t)$的相對應輸出 $y_1(t)$; (c)移位的輸入 $x_2(t) = x_1(t-2)$; (d) $x_2(t)$ 相對應的輸出 $y_2(t)$; (e)移位信號 $y_1(t-2)$。注意: 在 $y_2(t) \neq y_2(t-2)$ 時證明系統是時變的。

1.6.6 Linearity

- A ***linear*** system is a system that possesses the important property of ***superposition***: if an input consists of the weighted sum of several signals, then the output is the superposition — that is, the weighted sum — of the responses of the system to each of those signals.
- For example, the following system is ***linear*** if:

input	→	response output
$x(t)$	→	$y(t)$
$x_1(t)+x_2(t)$	→	$y_1(t)+y_2(t)$
$ax_1(t)+bx_2(t)$	→	$ay_1(t)+by_2(t)$

where a and b are any complex constants.

Examples 1.17, 1.18, 1.19

- $y(t)=tx(t)$ → linear
- $y(t)=x^2(t)$ → not linear
- $y(t)=Re\{x(t)\}$ → not linear

Example 1.20

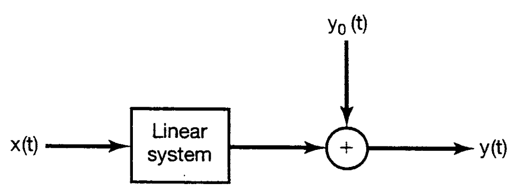

Figure 1.48 Structure of an incrementally linear system. Here, $y_0[n]$ is the zero-input response of the system.

- $y[n]=2x[n]+3$
- Responds ***linearly to changes in the input***
- That is, the difference between the responses to any two inputs is a ***linear*** function of the difference between two inputs.
- ***Incrementally*** linear systems

1.7 Summary

Through the chapter, we have learned about:

- Continuous-time signals and discrete-time signals
- Energy and power
- Odd and even signals
- Exponential and sinusoidal signal
- Periodical and non periodical signal
- Unit impulse and unit step function
- Time variance and time invariance
- Linear and nonlinear systems

The primary focus of this book will be on the class of linear, time-invariant system (LTI system.)