Signal and System Midterm

2012/11/13

班級: 姓名: 學號:

※請將計算過程寫在答案卷上，並於考試結束後繳回試卷與答案紙

共十題，滿分 100 分

1. A continuous-time signal is shown in Figure 1. Sketch and label carefully each of the following signals:

(a) $x(2t-1)$ (b) $x(-4-t/2)$

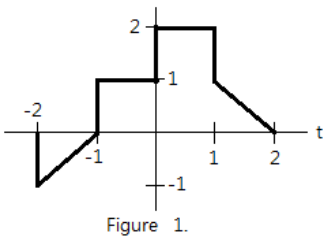

Figure 1.

2. Determine the values of P_∞ and E_∞ for each of the following signals:

(a) $x_2(t) = e^{j(2t+\pi/4)}$, (b) $x_3(t) = \cos(t)$,

(c) $x_2[n] = e^{j(\pi/2n+\pi/8n))}$ (d) $x_3[n] = \cos(\frac{\pi}{4}n)$

3. Determine the convolution of the following two signals:

$$x(t) = \begin{cases} t+1, & 0 \le t \le 1 \\ 2-t, & 1 < t \le 2 \\ 0, & elsewhere \end{cases}$$

$$h(t) = \delta(t+2) + 2\delta(t+1)$$

4. Consider an input $x[n]$ and a unit impulse response $h[n]$ given by

$$x[n] = \left(\frac{1}{3}\right)^{-n} u[-n-1],$$

$$h[n] = u[n-1]$$

Determine the output $y[n] = x[n] * h[n]$.

5. Consider the cascade of the following two systems S_1 and S_2, as depicted in Figure 2:

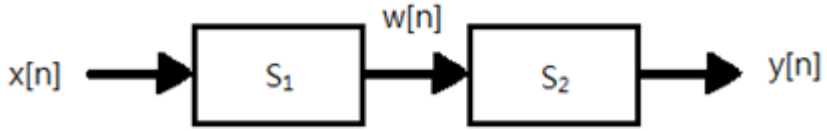

Figure 2.

S_1: causal LTI,

$$w[n] = \frac{1}{2}w[n-1] + x[n];$$

S_2: causal LTI,

$$y[n] = \alpha y[n-1] + \beta w[n];$$

The difference equation relating $x[n]$ and $y[n]$ is:

$$y[n] = -\frac{1}{8}y[n-2] + \frac{3}{4}y[n-1] + x[n]$$

Determine α and β and sketch the block diagram

6. Using the question 5 to show the impulse response of the cascade connection of S_1 and S_2.
7. For the continuous-time periodic signal

$$x(t) = 3 + 4\cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right),$$

Determine the fundamental frequency ω_0 and the Fourier series coefficients a_k

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

8. Suppose the periodic signal $x(t)$ has fundamental period T and Fourier coefficients a_k. In a variety of situations, it is easier to calculate the Fourier series coefficients b_k for $g(t) = \mathrm{d}x(t)/\mathrm{d}t$, as opposed to calculating a_k directly. Given that

$$\int_T^{2T} x(t)dt = 2,$$

Find an expression for a_k in terms of b_k and T.

9. Suppose we are given the following information about a signal $x(t)$:
   1. $x(t)$ is real and odd.
   2. $x(t)$ is periodic with period T=2 and has Fourier coefficients a_k .
   3. a_k =0 for $|k|$>1
   4. $\frac{1}{2}\int_0^2 |x(t)|^2 dt = 4.$

   Specify two different signals that satisfy these conditions.

10. Suppose we are given the following information about a signal $x[n]$:
    1. $x[n]$ is a real and even signal.
    2. $x[n]$ has period N=10 and Fourier coefficients a_k.
    3. a_{11}=5.
    4. $\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50.$

Show that $x[n]$=Acos(Bn+C), and specify numerical values for the constants A, B, C.