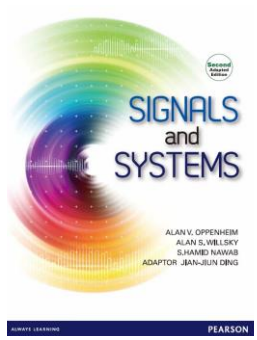
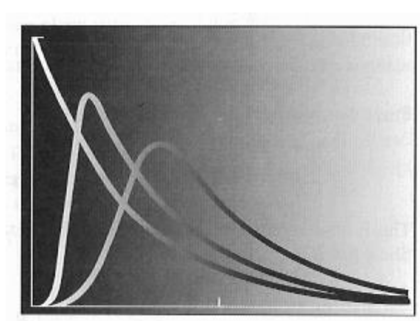
Signals and Systems



Alan. V. Oppenheim and Alan S. Willsky
with S. Hamid Nawab
Adaptor: Jian-Jiun Ding

Chapter 2

Linear Time Invariant Systems



Chapter 2 Linear Time Invariant Systems

Linear

if $x_1[n] \rightarrow y_1[n]$

$x_2[n] \rightarrow y_2[n]$

then $\alpha x_1[n] + \beta x_2[n] \rightarrow \alpha y_1[n] + \beta y_2[n]$

Time Invariant

if $x[n] \rightarrow y[n]$

then $x[n-n_0] \rightarrow y[n-n_0]$

Linear Time Invariant (LTI) system:
A system that is both linear and time-invariant.

Sec. 2.1 Discrete-time LTI Systems: The Convolution Sum

Key concepts
(i) definition of the discrete-time convolution;
(ii) unit impulse response;
(iii) ANY discrete-time LTI system can be modeled by a discrete-time convolution operation;
(iv) ANY discrete-time signals can be represented by a sum of impulses

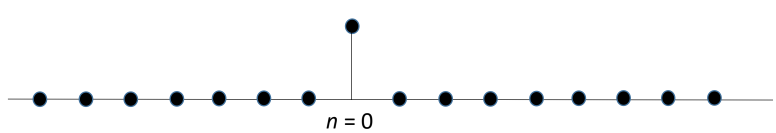
P.78

2.1.1 The Representation of Discrete-Time Signals in Terms of Impulses

Unit Impulses

$\delta[n] = 1$ when $n = 0$,

$\delta[n] = 0$ otherwise



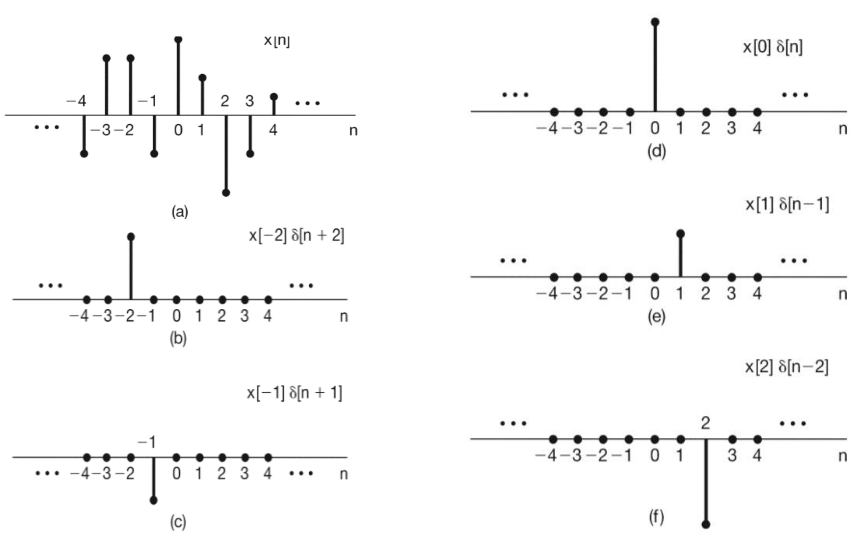
ANY discrete-time signals can be represented by a sum of impulses

$$x[n] = \ldots + x[-3]\delta[n+3] + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + x[2]\delta[n-2] + x[3]\delta[n-3] + \ldots$$

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k].$$

P.78

ANY discrete-time signals can be represented by a sum of impulses



P.79

2.1.2 The Discrete-Time Unit Impulse Response and the Convolution Sum Representation of LTI Systems

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k].$$

If the system is linear, then the output of the system corresponding to $x[n]$ can be expressed as:

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h_k[n]$$

where $h_k[n]$ is the output corresponding to $\delta[n-k]$.

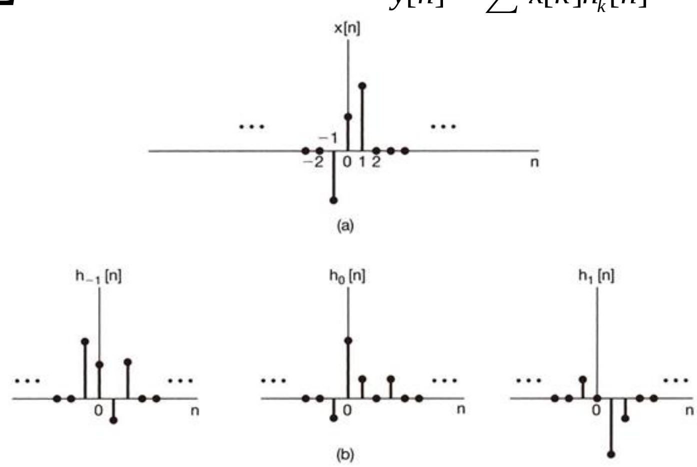
P.80

Input

$$x[n] = \sum_{k=}^{+\infty} x[k]\delta[n-k]$$

Output

$$y[n] = \sum^{+\infty} x[k]h_k[n]$$



A system that is linear but ***not*** time-invariant.

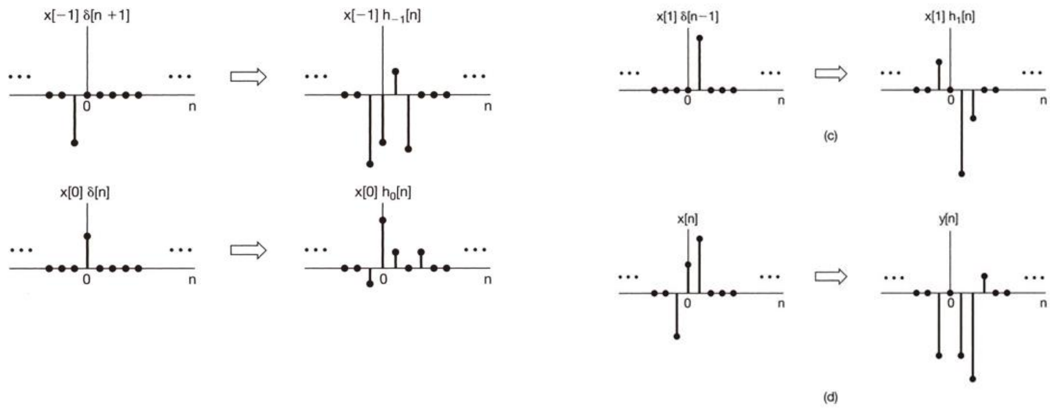
P.81

Input

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$

Output

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h_k[n]$$



A system that is linear but ***not*** time-invariant.

P.82

Input

$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\delta[n-k]$$

Output

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h_k[n]$$

If the system is time-invariant, then

$$h_k[n] = h_0[n-k]$$

Denoted $h_0[n]$ by $h[n]$, we have

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

This is the **Discrete-Time Convolution**. The convolution is usually denoted by $*$

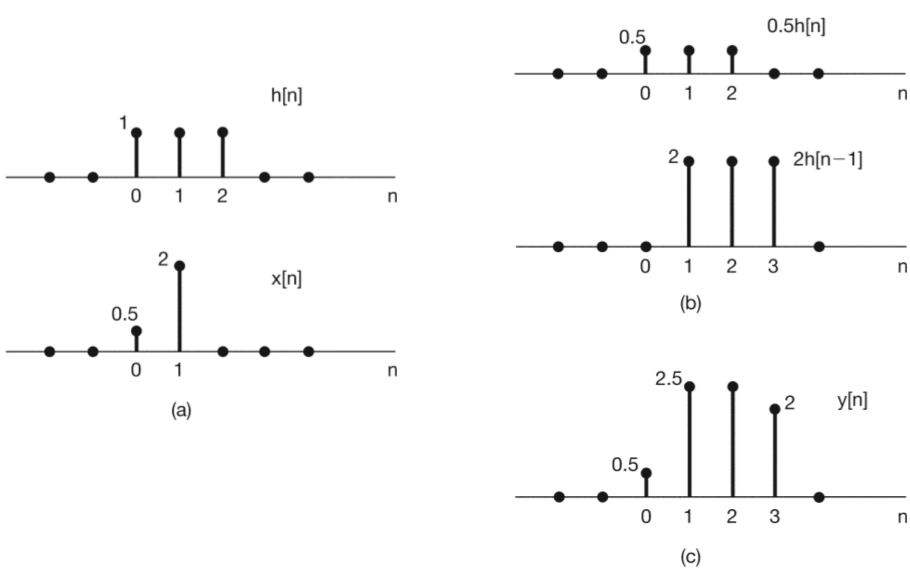
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$$

P.82

Important concept:

ANY LTI system can be expressed as a convolution operation.

[Example 2.1] $y[n]=x[n]*h[n]=x[0]h[n-0]+x[1]h[n-1]=0.5h[n]+2h[n-1].$



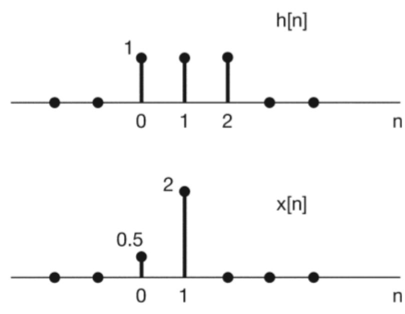
P.83

[Example 2.2]

(the same as Example 2.1, from different point of view)

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

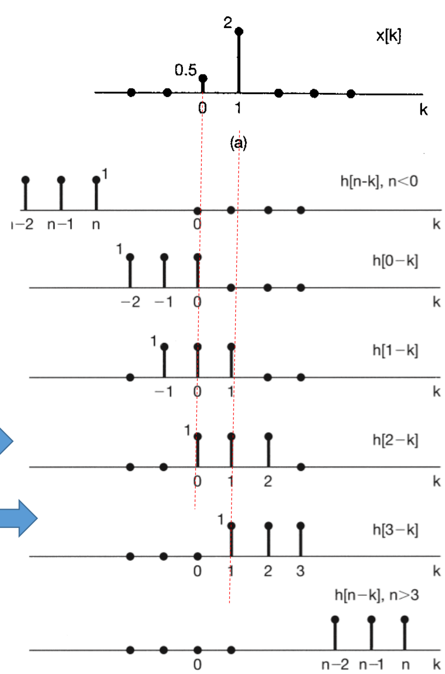
$y[n]$ is the sum of the product of $x[k]$ and $h[n\text{-}k]$.



(a)

P.84

[Example 2.2



(b)

n=0 $\quad y[0] = \sum_{k=-\infty}^{\infty} x[k]h[0-k] = 0.5.$

n=1 $\quad y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = 0.5 + 2.0 = 2.5.$

n=2 $\quad y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k] = 0.5 + 2.0 = 2.5.$

n=3 $\quad y[3] = \sum_{k=-\infty}^{\infty} x[k]h[3-k] = 2.0.$

$y[n] = 0$ otherwise

P.85

[Example 2.3]

$$x[n] = \alpha^n u[n],$$
$$h[n] = u[n],$$

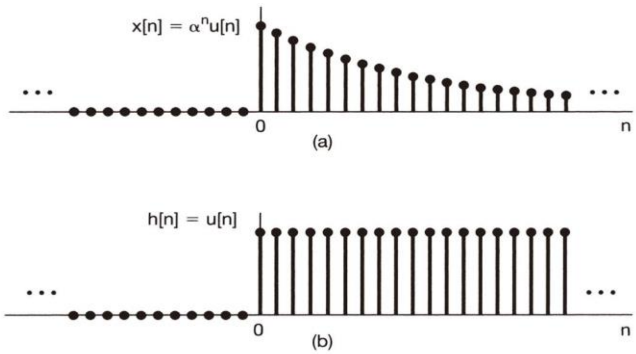


Figure 2.5 The signals $x[n]$ and $h[n]$ in Example 2.3.

P.86

[Example 2.3]

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

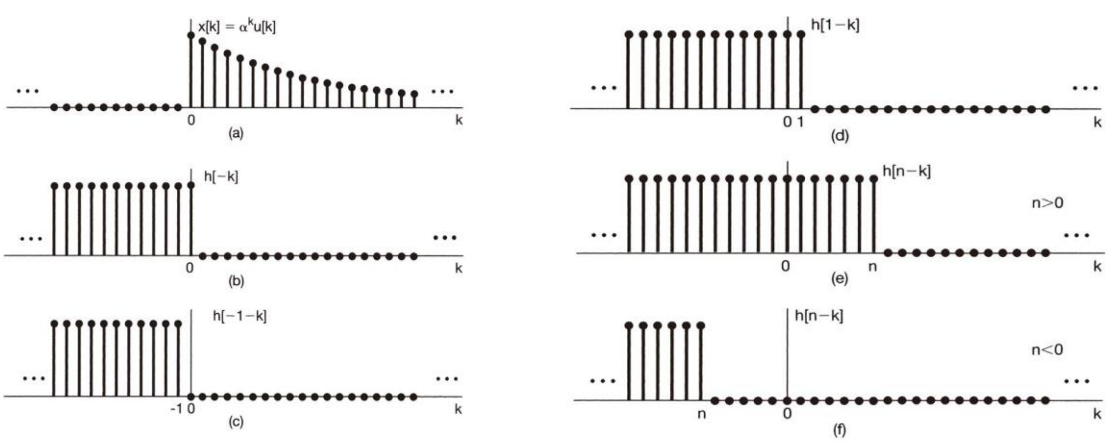


Figure 2.6 Graphical interpretation of the calculation of the convolution sum for Example 2.3.

P.87

[Example 2.3]

$x[n]=\alpha^n u[n],\qquad y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=\sum_{k=0}^{n}\alpha^k,$

$h[n]=u[n],$

$$y[n]=\left(\frac{1-\alpha^{n+1}}{1-\alpha}\right)u[n].$$

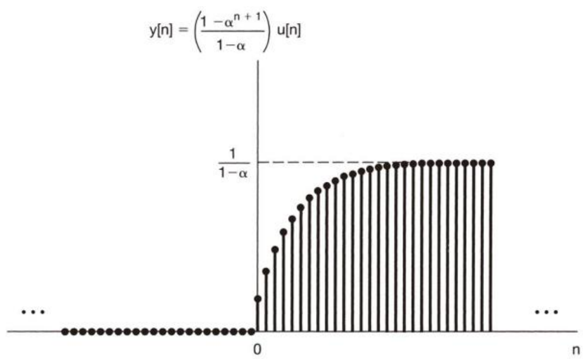


Figure 2.7 Output for Example 2.3.

P.88

[Example 2.4]

$$x[n]=\begin{cases}1, & 0\le n\le 4\\ 0, & otherwise\end{cases}\qquad h[n]=\begin{cases}\alpha^n, & 0\le n\le 6\\ 0, & otherwise\end{cases}.$$

$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$$

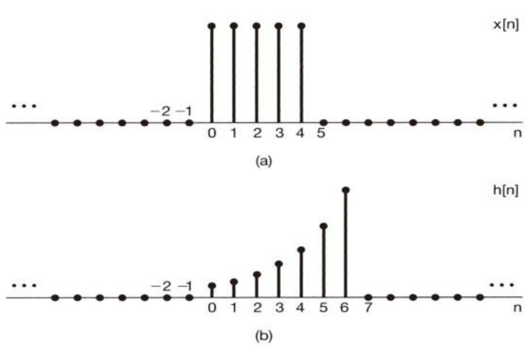
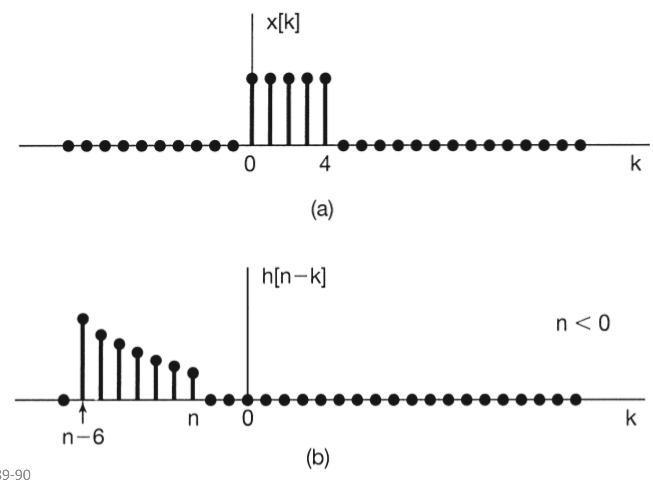


Figure 2.8 The signals to be convolved in Example 2.4.

P.88-89

[Example 2.4]

Interval 1: $n<0$

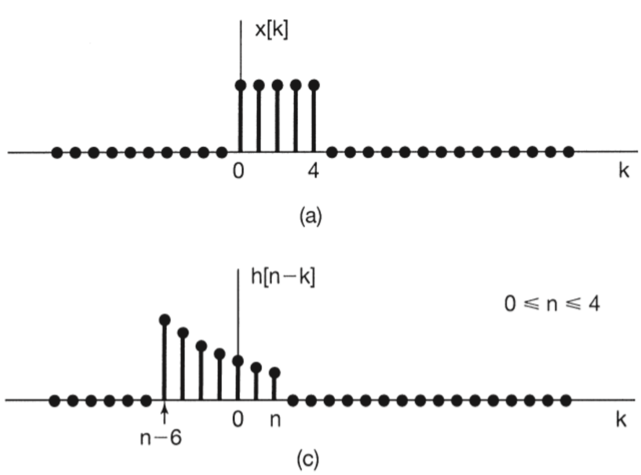


$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=0$$

P.89-90

[Example 2.4]

Interval 2: $0\le n\le 4$



$$y[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]=\sum_{k=0}^{n}\alpha^{n-k}.$$

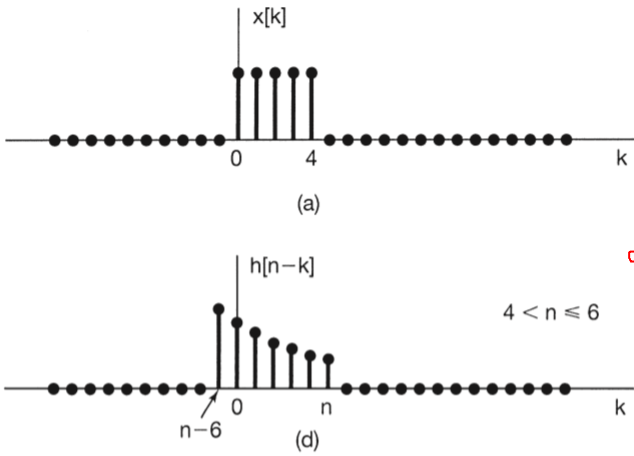
Let *n-k=r*
k=0 → *r=n*
k=n → *r=0*

$$y[n]=\sum_{r=0}^{n}\alpha^{r}=\frac{1-\alpha^{n+1}}{1-\alpha}.$$

P.89-90

[Example 2.4]

Interval 3: $4 < n \le 6$



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{4} \alpha^{n-k}.$$

$$y[n] = \alpha^{n} \sum_{k=0}^{4} (\alpha^{-1})^{k} = \frac{\alpha^{n-4} - \alpha^{n+1}}{1-\alpha}.$$

$$\alpha^{n} \cdot \frac{1-\alpha^{-5}}{1-\alpha^{-1}} = \frac{\alpha^{n} - \alpha^{n-5}}{1-\frac{1}{\alpha}}$$

$$= \frac{\alpha^{n+1} - \alpha^{n-4}}{\alpha - 1}$$

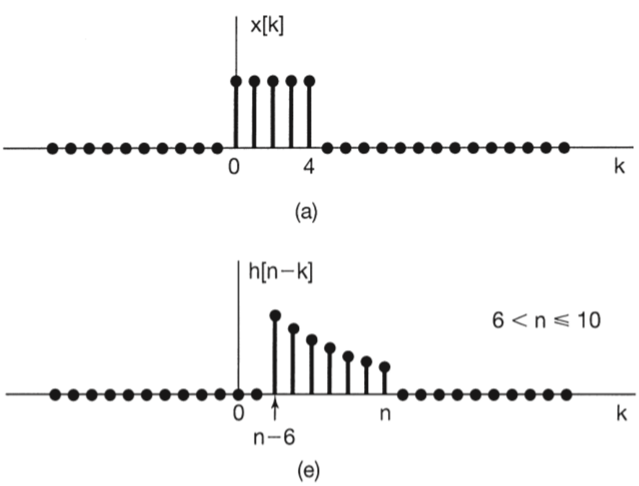
$$= \frac{\alpha^{n-4} - \alpha^{n+1}}{1-\alpha}$$

P.89-90

[Example 2.4]

Interval 4: $6 < n \le 10$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=n-6}^{4} \alpha^{n-k}.$$



Let *n-k=r*

k=n-6 → *r=6*

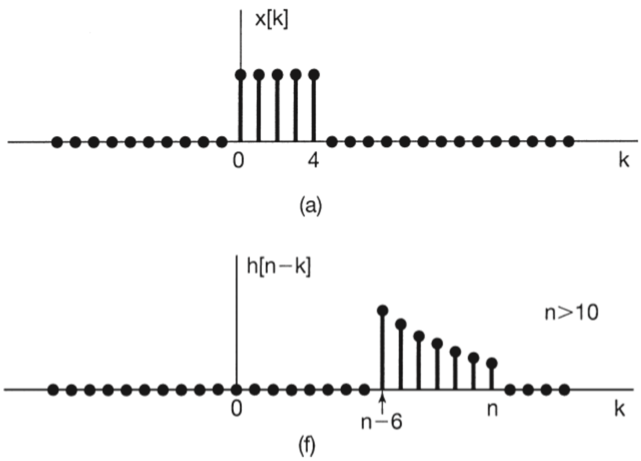
k=4 → *r=n-4*

$$y[n] = \sum_{r=6}^{n-4} \alpha^{r} = \sum_{r=0}^{10-n} \alpha^{6-r} = \frac{\alpha^{n-4} - \alpha^{7}}{1-\alpha}.$$

P.90-91

[Example 2.4]

Interval 5: $n > 10$



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = 0.$$

P.90-91

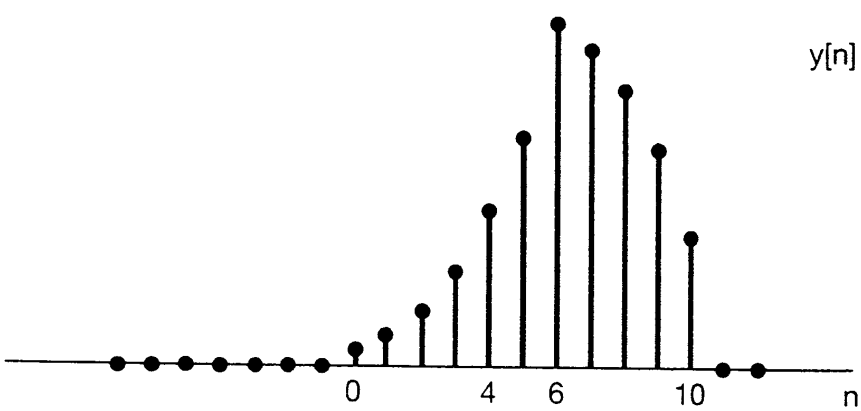


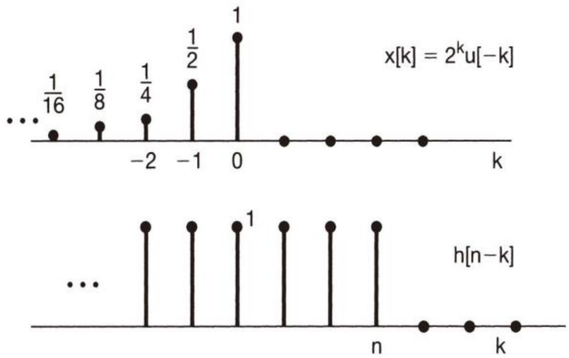
Figure 2.10 Result of performing the convolution in Example 2.4.

26

[Example 2.5]

$$x[n] = 2^n u[-n], \qquad h[n] = u[n].$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



(a)

Interval 1: $n \geq 0$

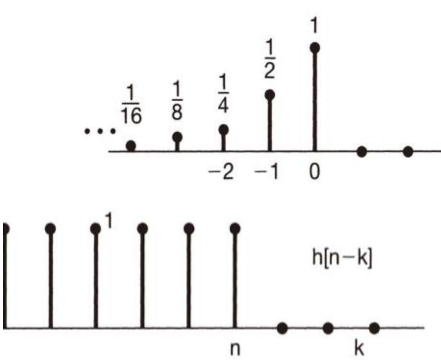
$$y[n] = \sum_{k=-\infty}^{0} x[k]h[n-k] = \sum_{k=-\infty}^{0} 2^k = 2$$

P.92-93

[Example 2.5]

$$x[n] = 2^n u[-n], \qquad h[n] = u[n].$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



(a)

Interval 2: $n < 0$

$$y[n] = \sum_{k=-\infty}^{n} 2^k = \sum_{l=-n}^{\infty} (\frac{1}{2})^l = \sum_{m=0}^{\infty} (\frac{1}{2})^{m-n}$$

$$= \left(\frac{1}{2}\right)^{-n} \sum_{m=0}^{\infty} (\frac{1}{2})^m = 2^n \cdot 2 = 2^{n+1}.$$

P.92-93

Sec. 2.2 Continuous-time LTI Systems: The Convolution Integral

Key concepts
(i) definition of the convolution integral;
(ii) ANY continuous-time LTI system can be modeled by the convolution integral

The concepts in this section can be viewed as the continuous counterpart of those in Section 2.1.

P.93

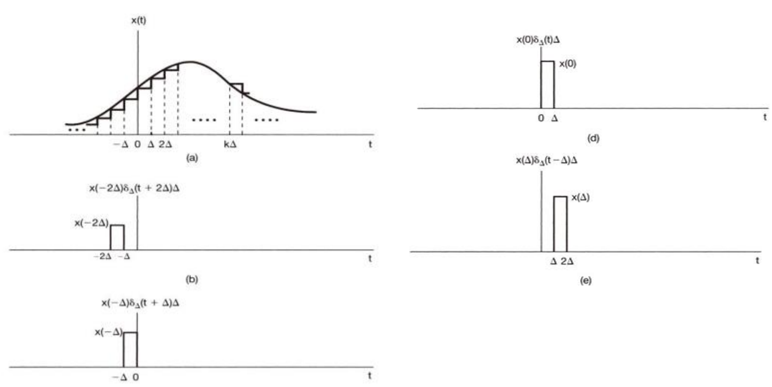
2.2.1 The Representation of Continuous-Time Signals in Terms of Impulses

A continuous function can be expressed as a linear combination of delayed unit pulses.

unit pulse: $$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta}, & 0 \le t \le \Delta \\ 0, & otherwise \end{cases},$$

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{+\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta.$$

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau.$$



P.94-95

$$x(t)=\int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau.$$

Specially, when $x(t) = u(t)$,

$$u(t)=\int_{0}^{\infty} \delta(t-\tau)d\tau.$$

2.2.2 The Continuous-Time Unit Impulse Response and the Convolution Integral Representation of LTI Systems

$$x(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{+\infty} x(k\Delta)\delta_{\Delta}(t-k\Delta)\Delta.$$

If a system is linear, when the input is $x(t)$, the corresponding output $y(t)$ can be expressed as:

$$y(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{+\infty} x(k\Delta)h_{k\Delta}(t)\Delta.$$

where $h_{k\Delta}(t)$ is the output corresponding to $\delta_{\Delta}(t-k\Delta)$

P.98

$$x(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{+\infty}x(k\Delta)\delta_{\Delta}(t-k\Delta)\Delta. \longrightarrow y(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{+\infty}x(k\Delta)h_{k\Delta}(t)\Delta$$

$$=\int_{-\infty}^{+\infty}x(\tau)h_{\tau}(t)d\tau.$$

setting $\tau=k\Delta$

Furthermore, if a system is linear and time-invariant, then

$$h_{\tau}(t)=h_0(t-\tau)$$

For notational convenience, we use $h(t)$ to denote $h_0(t)$

$$\boxed{y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau.}$$

P.98-100

$$x(t)\xrightarrow{\text{LTI system}}y(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau.$$

continuous-time convolution:

$$y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau.$$

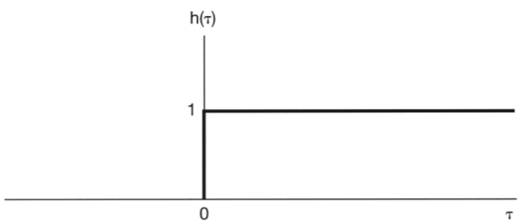
$h(t)$: unit impulse response
(impulse response)
i.e., the output of the system when the input is $\delta(t)$

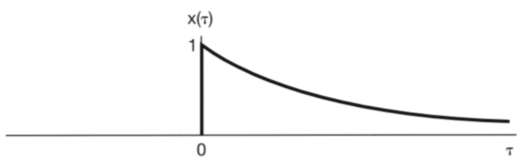
P.98-100

[Example 2.6]

$$x(t) = e^{-at}u(t), \quad a > 0 \qquad h(t) = u(t).$$

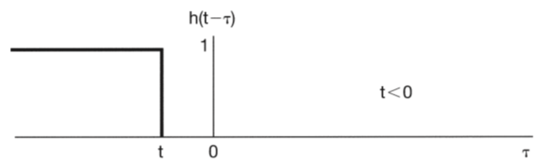
$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau.$$





Interval 1: $t < 0$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = 0.$$

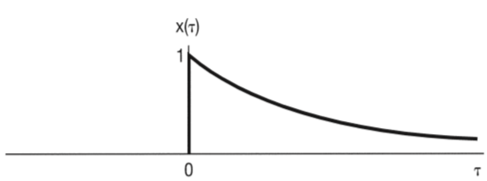


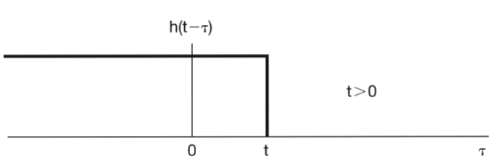
P.101-102

[Example 2.6]

$$x(t) = e^{-at}u(t), \quad a > 0 \qquad h(t) = u(t).$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau.$$





Interval 2: $t > 0$

$$x(\tau)h(t-\tau) = \begin{cases} e^{-a\tau}, & 0 < \tau < t \\ 0, & otherwise \end{cases}.$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_{0}^{t} e^{-a\tau} d\tau$$

$$= \frac{1}{a}(1 - e^{-at}).$$

P.101-102

[Example 2.6]

$$x(t) = e^{-at}u(t), \quad a > 0 \qquad\qquad h(t) = u(t).$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau.$$

$$y(t) = \frac{1}{a}(1-e^{-at}) \quad \text{for } t > 0$$

$$y(t) = 0 \qquad\qquad \text{for } t < 0$$

$$y(t) = \frac{1}{a}(1-e^{-at})u(t)$$

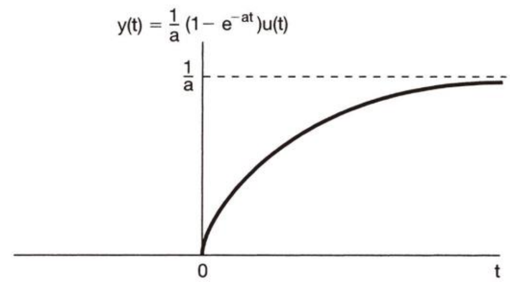


Figure 2.18 Response of the system in Example 2.6 with impulse response $h(t) = u(t)$ to the input $x(t) = e^{-at}u(t)$.

P.101-102

Sec. 2.3 Properties of Linear Time-Invariant Systems

Key concepts

(i) All of the LTI systems have the following properties: (a) linearity, (b) time invariance, (c) the commutative property, (d) the distributive property, and (e) the associative property.

(ii) Moreover, some of the LTI systems have the properties of (a) memory (or memoryless), (b) invertibility, (c) causality, and (d) stability.

(iii) Learn the definitions of (a) absolutely summable, (b) absolutely integrable, and (c) the unit step response.

(iv) Learn the change of the support after convolution.

P.106

Discrete-Time Linear Time-Invariant (LTI) System

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = x[n]*h[n]$$

Continuous-Time Linear Time-Invariant (LTI) System

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = x(t)*h(t)$$

[Example 2.9]

$$h[n] = \begin{cases} 1, & n = 0,1 \\ 0, & otherwise \end{cases}. \qquad y[n] = x[n]*h[n] = x[n] + x[n-1]$$

The following systems have the same impulse response (the same response when $x[n] = \delta[n]$) but not LTI.

$$y[n] = (x[n] + x[n-1])^2,$$

$$y[n] = \max(x[n], x[n-1]).$$

P.106-107

2.3.1 The Commutative Property

Discrete-Time

$$x[n]*h[n] = h[n]*x[n]$$

$$\sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k],$$

Continuous-Time

$$x(t)*h(t) = h(t)*x(t)$$

$$\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau.$$

P.107

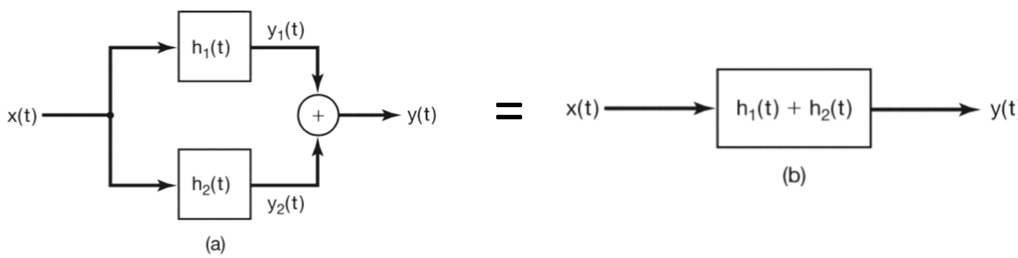
2.3.2 The Distributive Property

Discrete-Time

$$x[n]*(h_1[n]+h_2[n]) = x[n]*h_1[n]+x[n]*h_2[n],$$

Continuous-Time

$$x(t)*(h_1(t)+h_2(t)) = x(t)*h_1(t)+x(t)*h_2(t)$$



P.108

[Example 2.10]

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n], \qquad h[n] = u[n].$$

$$y[n] = x[n]*h[n] = y_1[n]+y_2[n]$$

$$y_1[n] = \left(\frac{1}{2}\right)^n u[n]*h[n] \qquad y_2[n] = 2^n u[-n]*h[n]$$

P.109-110

2.3.3 The Associative Property

Discrete-Time

$$x[n]*(h_1[n]*h_2[n])=(x[n]*h_1[n])*h_2[n]$$

Continuous-Time

$$x(t)*[h_1(t)*h_2(t)]=[x(t)*h_1(t)]*h_2(t)$$

P.110-111

2.3.4 LTI Systems with and without Memory

Discrete-Time

memoryless $y[n]=Kx[n]$

i.e., $h[n]=0$ when $n\neq 0$

Otherwise, the system has memory.

Continuous-Time

memoryless $y(t)=Kx(t)$

i.e., $h(t)=0$ when $t\neq 0$

Otherwise, the system has memory.

P.112-113

2.3.5 Invertibility of LTI Systems

Discrete-Time

If $h[n]$ is the impulse response of a discrete LTI system, then the system has the reversibility property if and only if there exists an $h_1[n]$ such that

$$h[n] * h_1[n] = \delta[n]$$

Continuous-Time

If $h(t)$ is the impulse response of a continuous LTI system, then the system has the reversibility property if and only if there exists an $h_1(t)$ such that

$$h(t) * h_1(t) = \delta(t)$$

P.114

[Example 2.11]

$$y(t) = x(t) * h(t) = x(t - t_0)$$

$$h(t) = \delta(t - t_0)$$

If

$$\boxed{h_1(t) = \delta(t + t_0)}$$

$$y(t) * h_1(t) = y(t + t_0) = x(t)$$

$$h(t) * h_1(t) = \delta(t)$$

P.115

[Example 2.12]

If $h[n] = u[n]$

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]u[n-m] = \sum_{m=-\infty}^{n} x[m]$$

When

$$h_1[n] = \delta[n] - \delta[n-1]$$

$$y[n] * h_1[n] = y[n] - y[n-1] = x[n]$$

$$h[n] * h_1[n] = \delta[n]$$

P.115-116

2.3.6 Causality for LTI Systems

Discrete-Time

$$h[n] = 0 \qquad \text{for } n < 0$$

$$y[n] = \sum_{k=-\infty}^{n} x[k]h[n-k] = \sum_{k=0}^{\infty} h[k]x[n-k],$$

Continuous-Time

$$h(t) = 0 \qquad \text{for } t < 0$$

$$y(t) = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau$$

P.116-117

2.3.7 Stability for LTI Systems

Discrete-Time

If $|x[n]|$ is bounded, then $|y[n]|$ is also bounded.

Sufficient condition for a discrete-time LTI system to be stable

$$\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$$

Continuous-Time

If $|x(t)|$ is bounded, then $|y(t)|$ is also bounded.

Sufficient condition for a continuous-time LTI system to be stable

$$\int_{-\infty}^{+\infty} |h(\tau)|\, d\tau < \infty$$

P.117-118

[Example 2.13]

$h[n] = \delta[n]$

$h(t) = \delta(t)$

→ stable

$h[n] = u[n]$

$h(t) = u(t)$

→ may not be stable

P.119

2.3.8 The Unit Step Response of an LTI System

Unit step response: The response when the input is $u[n]$ (or $u(t)$)

Discrete-Time

The unit step response $s[n]$ is

$$s[n] = u[n] * h[n] = \sum_{k=-\infty}^{n} h[k]$$

Therefore, $h[n] = s[n] - s[n-1]$

Continuous-Time

$$s(t) = u(t) * h(t) = \int_{-\infty}^{t} h(\tau) d\tau,$$

Therefore, $h(t) = \dfrac{ds(t)}{dt} = s'(t)$

P.120-121

2.3.9 Variation of Support and Length After Convolution

Support: A set of points where a function is nonzero.

Continuous-time case

If

$x(t) = 0$ for $t < t_1$ and $t > t_2$, $t_2 > t_1$,

$x(t) \neq 0$ for $t_1 < t < t_2$,

support: $t \in (t_1, t_2)$

length: $t_2 - t_1$.

P.121

Support and Length Variation Property for Continuous-Time Convolution

If the support of $x(t)$ is $t \in (t_1, t_2)$
the support of $h(t)$ is $t \in (t_3, t_4)$

$$y(t) = x(t) * h(t)$$

then the support of $y(t)$ is equal to (or within)

$$t \in (t_1 + t_3,\ t_2 + t_4)$$

the length of $y(t)$ is

$$L_y = t_2 + t_4 - t_3 - t_1 = L_x + L_h.$$

P.121

Discrete-time case

If

$x[n] = 0$ for $t < n_1$ and $t > n_2$, $n_2 > n_1$,

$x[n] \neq 0$ for $n_1 < t < n_2$,

support: $n \in [n_1, n_2]$

length: $n_2 - n_1 + 1$

P.122

Support and Length Variation Property for Discrete-Time Convolution

If the support of $x[n]$ is $n \in [n_1, n_2]$
the support of $h[n]$ is $n \in [n_3, n_4]$

$$y[n] = x[n] * h[n]$$

then the support of $y[n]$ is equal to (or within)

$$n \in [n_1 + n_3,\ n_2 + n_4]$$

the length of $y[n]$ is

$$L_y = n_2 + n_4 - n_3 - n_1 + 1 = L_x + L_h - 1.$$

P.122-123

Sec. 2.4 Causal LTI Systems Described by Differential and Difference Equations

Key concepts

(i) when the initial conditions are all zero, a linear differential / difference equation is a linear system.
(ii) with the condition of initial rest, a linear differential / difference equation with constant coefficients is a linear time-invariant (LTI) system.
(iii) how to use block diagrams to represent a system

P.123

2.4.1 Linear Constant-Coefficient Differential Equations

[Example 2.14]

$$\frac{dy(t)}{dt} + 2y(t) = x(t) \quad \text{where} \quad x(t) = Ke^{3t}u(t)$$

Solution:

$$y(t) = y_p(t) + y_h(t)$$

$y_h(t)$ is the solution of $\frac{dy(t)}{dt} + 2y(t) = 0$

$y_p(t)$ is any the original solution

$$y_h(t) = Ae^{st} \qquad y_p(t) = \frac{K}{5}e^{3t}, \quad t > 0$$

P.125

A Linear Constant Coefficient Differential Equation with Initial Rest is Causal and LTI

If

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

and the system in initial rest

$$y(t_0) = \frac{dy(t_0)}{dt} = \ldots = \frac{d^{N-1}y(t_0)}{dt^{N-1}} = 0$$

then the system is causal and LTI.

P.127-128

2.4.2 Linear Constant-Coefficient Difference Equations

A Linear Constant Coefficient Difference Equation with Initial Rest Is Causal and LTI

If
$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

and the system in initial rest

$$y[n_0-1] = y[n_0-2] = \ldots = y[n_0-N] = 0.$$

then the system is causal and LTI.

P.128

2.4.3 Block Diagram Representations of First-Order Systems Described by Differential and Difference Equations

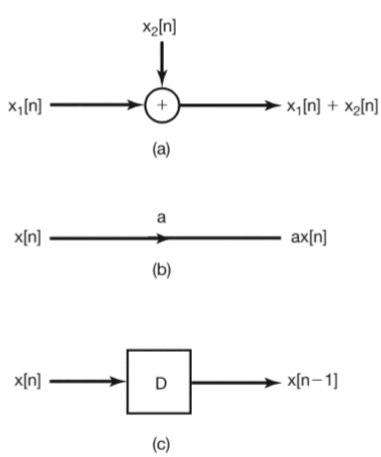


Figure 2.27 Basic elements for the block diagram representation of the causal system described by eq. (2.126): (a) an adder; (b) multiplication by a coefficient; (c) a unit delay.

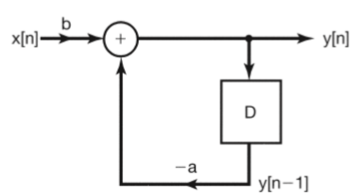


Figure 2.28 Block diagram representation for the causal discrete-time system described by eq. (2.126).

P.133

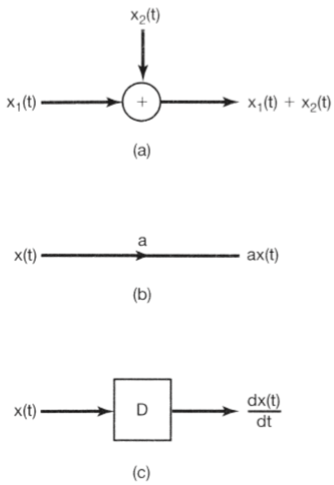


Figure 2.29 One possible set of basic elements for the block diagram representation of the continuous-time system described by eq. (2.128): (a) an adder; (b) multiplication by a coefficient; (c) a differentiator.

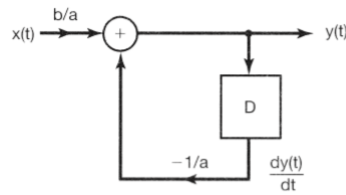


Figure 2.30 Block diagram representation for the system in eqs. (2.128) and (2.129), using adders, multiplications by coefficients, and differentiators.

P.134

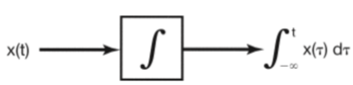


Figure 2.31 Pictorial representation of an integrator.

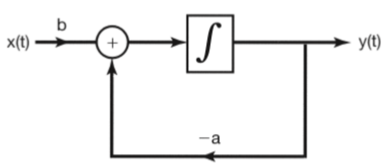


Figure 2.32 Block diagram representation for the system in eqs. (2.128) and (2.131), using adders, multiplications by coefficients, and integrators.

$$y(t) = \int_{-\infty}^{t} \left[bx(\tau) - ay(\tau) \right] d\tau. \tag{2.131}$$

P.135

Sec. 2.5 Singularity Functions

Key concepts

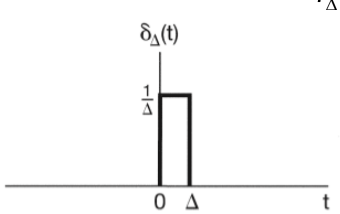
(i) studying the property of the continuous unit impulse (summarized in TA Table 2.1);
(ii) studying the unit doublet and its property

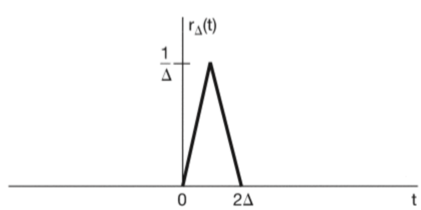
P.135

2.5.1 The Unit Impulse as an Idealized Short Pulse

$x(t) = x(t) * \delta(t)$ when $x(t) = \delta(t)$ $\delta(t) = \delta(t) * \delta(t)$

$$r_\Delta(t) = \delta_\Delta(t) * \delta_\Delta(t)$$





$\delta_\Delta(t)\big|_{\Delta\to 0}$ and $r_\Delta(t)\big|_{\Delta\to 0}$ can all be viewed as a unit impulse.

There is no explicit form of a unit impulse.
Instead, we can say some function behaves like a unit impulse

P.136

2.5.2 Defining the Unit Impulse through Convolution

We define $\delta(t)$ as the signal for which

$$x(t) = x(t) * \delta(t)$$

is satisfied.

P.137

2.5.3 Unit Doublets and Other Singularity Functions

Definition 2.6 Unit Doublet

$$u_1(t) = \frac{d}{dt}\delta(t) \qquad \frac{d}{dt}x(t) = x(t) * u_1(t)$$

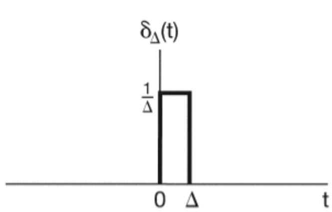


Figure 1.34

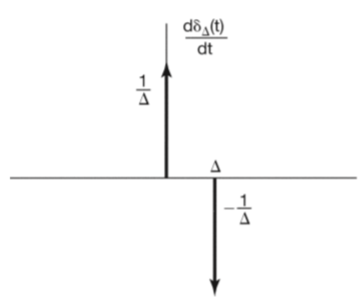


Figure 2.36 The derivative $d\delta_\Delta(t)/dt$ of the short rectangular pulse $\delta_\Delta(t)$ of Figure 1.34.

P.141-143

Self-convolution of the Unit Doublet

$$u_2(t) = u_1(t) * u_1(t) = \frac{d^2}{dt^2}\delta(t) \qquad x(t) * u_2(t) = \frac{d^2}{dt^2}x(t)$$

$$u_k(t) = \underbrace{u_1(t) * \cdots * u_1(t)}_{k\ times} = \frac{d^k}{dt^k}\delta(t) \qquad x(t) * u_k(t) = \frac{d^k}{dt^k}x(t)$$

P.141-142

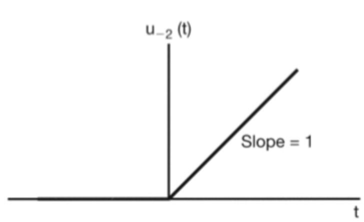
Unit Step Function

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau, \qquad x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Self-Convolution of the Step Function

$$u_{-2}(t) = u(t) * u(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = tu(t).$$ (unit ramp function)

$$x(t) * u_{-2}(t) = x(t) * u(t) * u(t) = \int_{-\infty}^{t} \left(\int_{-\infty}^{\tau} x(\sigma)\, d\sigma \right) d\tau.$$



P.143-144

Self-Convolution of the Step Function

$$u_{-k}(t) = \underbrace{u(t) * \cdots * u(t)}_{k\ times} = \int_{-\infty}^{t} u_{-(k-1)}(\tau)d\tau = \frac{t^{k-1}}{(k-1)!}u(t)$$

$$x(t) * u_{-k}(t) = \int_{-\infty}^{t}\int_{-\infty}^{\tau_{k-1}}\cdots\int_{-\infty}^{\tau_2}\left(\int_{-\infty}^{\tau_1} x(\sigma)d\sigma\right)d\tau_1 d\tau_2 \cdots d\tau_{k-1}$$

$$\delta(t) = u_0(t),$$
$$u(t) = u_{-1}(t).$$

$$u_k(t) * u_r(t) = u_{k+r}(t)$$

P.144

Ta Table 2.1

Properties of the Continuous Unit Impulse and Other Singularity Functions

Property or Definition	Formula
(1) Integration	$\int_{-\infty}^{\infty} \delta(t)dt = 1$
(2) Relation with the unit step function	$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$, $\frac{d}{dt}u(t) = \delta(\tau)$
(3) Convolution	$x(t) * \delta(t) = x(t)$
(4) Auto convolution	$\delta(t) * \delta(t) = \delta(t)$, $\delta(t) * \delta(t) * .... * \delta(t) = \delta(t)$
(5) Sifting (I)	$\int_{a}^{b} f(t)\delta(t - t_0)dt = f(t_0)$ if $a < t_0 < b$
(6) Sifting (II)	$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$
(7) Unit doublet $u_1(t)$	$u_1(t) = \frac{d}{dt}\delta(t)$ $x(t) * u_1(t) = \frac{d}{dt}x(t)$
(8) $u_k(t)$ (k is a positive integer)	$u_k(t) = \underbrace{u_1(t) * \cdots * u_1(t)}_{k \text{ times}} = \frac{d^k}{dt^k}\delta(t)$ $x(t) * u_k(t) = \frac{d^k}{dt^k}x(t)$
(9) $u_{-1}(t)$	$u_{-1}(t) = u(t)$,
(10) $u_{-k}(t)$ (k is a positive integer)	$u_{-k}(t) = \underbrace{u(t) * \cdots * u(t)}_{k \text{ times}} = \frac{t^{k-1}}{(k-1)!}u(t)$, $x(t) * u_{-k}(t) = \int_{-\infty}^{t}\int_{-\infty}^{\tau_{k-1}} \cdots \int_{-\infty}^{\tau_2}\left(\int_{-\infty}^{\tau_1} x(\sigma)d\sigma\right)d\tau_1 d\tau_2 \ldots d\tau_{k-1}$. ($k$ times of integration)
When $k = 2$, it is called a unit ramp function	

P.146

Sec. 2.6 LTI Systems in the Multiple Dimensional Case

Key concepts

Learning
(i) the LTI system in the multiple dimensional case,
(ii) the impulse response in the multiple dimensional case,
(iii) the convolution operation in the multiple dimensional case,
(iv) how the range varies after performing multiple dimensional convolution

P.145

Multiple dimensional system

$$x(t_1,t_2,\cdots,t_N)\rightarrow y(t_1,t_2,\cdots,t_N)$$

Linear Multiple dimensional system

$$\alpha x_1(t_1,t_2,\cdots,t_N)+\beta x_2(t_1,t_2,\cdots,t_N)\rightarrow \alpha y_1(t_1,t_2,\cdots,t_N)+\beta y_2(t_1,t_2,\cdots,t_N)$$

if $x_1(t_1,t_2,\cdots,t_N)\rightarrow y_1(t_1,t_2,\cdots,t_N)$ and $x_2(t_1,t_2,\cdots,t_N)\rightarrow y_2(t_1,t_2,\cdots,t_N)$

Time-invariant multiple dimensional system

$$x(t_1-d_1,t_2-d_2,\cdots,t_N-d_N)\rightarrow y(t_1-d_1,t_2-d_2,\cdots,t_N-d_N)$$

P.145-146

A multiple dimensional linear and time-invariant (LTI) system can be expressed as a convolution form:

$$y(t_1,t_2,\cdots,t_N)=\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(\tau_1,\tau_2,\cdots,\tau_N)h(t_1-\tau_1,t_2-\tau_2,\cdots,t_N-\tau_N)d\tau_1 d\tau_2\cdots d\tau_N$$

where $h(\tau_1,\tau_2,\cdots,\tau_N)$ is the response when the input is a multiple dimensional unit impulse:

$$\delta(t_1,t_2,\cdots,t_N)\rightarrow h(t_1,t_2,\cdots,t_N)$$

where $\delta(t_1,t_2,\cdots,t_N)=\delta(t_1)\delta(t_2)\cdots\delta(t_N)$

P.147

Multiple dimensional system

$$x[n_1,n_2,\cdots,n_N]\rightarrow y[n_1,n_2,\cdots,n_N]$$

Linear Multiple dimensional system

$$\alpha x_1[n_1,n_2,\cdots,n_N]+\beta x_2[n_1,n_2,\cdots,n_N]\rightarrow \alpha y_1[n_1,n_2,\cdots,n_N]+\beta y_2[n_1,n_2,\cdots,n_N]$$

if $x_1[n_1,n_2,\cdots,n_N]\rightarrow y_1[n_1,n_2,\cdots,n_N]$ and $x_2[n_1,n_2,\cdots,n_N]\rightarrow y_2[n_1,n_2,\cdots,n_N]$

Time-invariant multiple dimensional system

$$x[n_1-d_1,n_2-d_2,\cdots,n_N-d_N]\rightarrow y[n_1-d_1,n_2-d_2,\cdots,n_N-d_N]$$

P.147

A multiple dimensional linear and time-invariant (LTI) system can be expressed as a convolution form:

$$y[n_1, n_2, \cdots, n_N] = \sum_{k_N=-\infty}^{\infty} \cdots \sum_{k_2=-\infty}^{\infty} \sum_{k_1=-\infty}^{\infty} x[k_1, k_2, \cdots, k_N] h[n_1 - k_1, n_2 - k_2, \cdots, n_N - k_N]$$

where $h[n_1, n_2, \ldots, n_N]$ is the response when the input is a multiple dimensional unit impulse:

$$\delta[n_1, n_2, \cdots, n_N] \rightarrow h[n_1, n_2, \cdots, n_N]$$

where $\delta[n_1, n_2, \cdots, n_N] = \delta[n_1]\delta[n_2]\cdots\delta[n_N]$

P.148

[Example 2.17]

$x[n_1, n_2] = 255$ for $|n_1| \leq 3$ and $|n_2| \leq 3$, $x[n_1, n_2] = 0$ otherwise.

$h[-1, 0] = h[-1, -1] = h[0, -1] = 1$,

$h[1, 0] = h[1, 1] = h[0, 1] = -1$, $h[n_1, n_2] = 0$ otherwise

$$y[n_1, n_2] = \sum_{k_2=-\infty}^{\infty} \sum_{k_1=-\infty}^{\infty} x[k_1, k_2] h[n_1 - k_1, n_2 - k_2]$$

P.148

[Example 2.17]

(a) $x[n_1, n_2]$

	$n_2=-4$	-3	-2	-1	0	1	2	3	4
$n_1=-4$	0	0	0	0	0	0	0	0	0
$n_1=-3$	0	255	255	255	255	255	255	255	0
$n_1=-2$	0	255	255	255	255	255	255	255	0
$n_1=-1$	0	255	255	255	255	255	255	255	0
$n_1=0$	0	255	255	255	255	255	255	255	0
$n_1=1$	0	255	255	255	255	255	255	255	0
$n_1=2$	0	255	255	255	255	255	255	255	0
$n_1=3$	0	255	255	255	255	255	255	255	0
$n_1=4$	0	0	0	0	0	0	0	0	0

(b) $h[n_1, n_2]$

	$n_2=-2$	-1	0	1	2
$n_1=-2$	0	0	0	0	0
$n_1=-1$	0	2	1	0	0
$n_1=0$	0	1	0	−1	0
$n_1=1$	0	0	−1	−2	0
$n_1=2$	0	0	0	0	0

$$y[n_1,n_2]=\sum_{k_2=-\infty}^{\infty}\sum_{k_1=-\infty}^{\infty}x[k_1,k_2]h[n_1-k_1,n_2-k_2]$$

(c) $y[n_1, n_2]$

	$n_2=-5$	-4	-3	-2	-1	0	1	2	3	4	5
$n_1=-5$	0	0	0	0	0	0	0	0	0	0	0
$n_1=-4$	0	510	765	765	765	765	765	765	255	0	0
$n_1=-3$	0	765	1020	765	765	765	765	765	0	−255	0
$n_1=-2$	0	765	765	0	0	0	0	0	−765	−765	0
$n_1=-1$	0	765	765	0	0	0	0	0	−765	−765	0
$n_1=0$	0	765	765	0	0	0	0	0	−765	−765	0
$n_1=1$	0	765	765	0	0	0	0	0	−765	−765	0
$n_1=2$	0	765	765	0	0	0	0	0	−765	−765	0
$n_1=3$	0	255	0	−765	−765	−765	−765	−765	−1020	−765	0
$n_1=4$	0	0	−255	−765	−765	−765	−765	−765	−765	−510	0
$n_1=5$	0	0	0	0	0	0	0	0	0	0	0

P.149

[Properties]

Except for that causality, memory/memoryless, and the unit step function are hard to define in the multiple dimensional case, other properties listed in Section 2.3 can also be applied to the multiple dimensional case.

stable

$$\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|h(\tau_1,\tau_2,\cdots,\tau_N)\right|d\tau_1 d\tau_2\cdots d\tau_N<\infty$$

$$\sum_{k_N=-\infty}^{\infty}\cdots\sum_{k_2=-\infty}^{\infty}\sum_{k_1=-\infty}^{\infty}\left|h[k_1,k_2,\cdots,k_N]\right|<\infty$$

P.150

[Support and Size]

If $x(t_1, t_2, \cdots, t_N) = 0$ for $t_1 < a_1$ or $t_1 > b_1$, $t_2 < a_2$ or $t_2 > b$, …………,
$t_N < a_N$ or $t_N > b_N$

$x(t_1, t_2, \cdots, t_N) \neq 0$ for $a_1 < t_1 < b_1$, $a_2 < t_2 < b_2$, ……, $a_N < t_1 < b_N$.

support:

$$\{(t_1, t_2, \ldots, t_N) \,|t_1 \in (a_1, b_1), \quad t_2 \in (a_2, b_2), \quad \ldots.., \quad t_N \in (a_N, b_N)\}$$

size:

$$S_1 \times S_2 \times \ldots. \times S_N \quad \text{where } S_n = b_n - a_n, \quad n = 1, 2, \ldots, N.$$

P.150

[Support and Size after Convolution]

Suppose that

the support of $x(t_1, t_2, \ldots, t_N)$ is $\{(t_1, t_2, \ldots, t_N) \,|t_1 \in (a_1, b_1), t_2 \in (a_2, b_2), \ldots.., t_N \in (a_N, b_N)\}$

the support of $h(t_1, t_2, \ldots, t_N)$ is $\{(t_1, t_2, \ldots, t_N) \,|t_1 \in (c_1, d_1), t_2 \in (c_2, d_2), \ldots.., t_N \in (c_N, d_N)\}$.

If

$$y(t_1, t_2, \cdots, t_N) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau_1, \tau_2, \cdots, \tau_N) h(t_1 - \tau_1, t_2 - \tau_2, \cdots, t_N - \tau_N) d\tau_1 d\tau_2 \cdots d\tau_N$$

the support of $y(t_1, t_2, \ldots, t_N)$ is

$$\{(t_1, t_2, \ldots, t_N) \,|t_1 \in (a_1+c_1, b_1+d_1), t_2 \in (a_2+c_2, b_2+d_2), \quad \ldots.., \quad t_N \in (a_N+c_N, b_N+d_N)\}.$$

the size of $y(t_1, t_2, \ldots, t_N)$ is

$$(S_1 + T_1) \times (S_2 + T_2) \times \ldots. \times (S_N + T_N)$$

$(S_n = b_n - a_n)$

$(T_n = d_n - c_n)$

P.151

[Support and Size]

If $x[n_1,n_2,\cdots,n_N]=0$ for $n_1 < a_1$ or $n_1 > b_1$, $n_2 < a_2$ or $n_2 > b$, …………,
$n_N < a_N$ or $n_N > b_N$

$x[n_1,n_2,\cdots,n_N]\neq 0$ for $a_1 < n_1 < b_1$, $a_2 < n_2 < b_2$, ……, $a_N < n_1 < b_N$.

support:

$\{(n_1, n_2, \ldots, n_N)\ |n_1 \in [a_1,\ b_1],\ \ n_2 \in [a_2,\ b_2],\ \ \ldots..,\ \ n_N \in [a_N,\ b_N]\}$.

size:

$S_1 \times S_2 \times \ldots. \times S_N$ where $S_n = b_n - a_n+1$, $n = 1, 2, \ldots, N$.

P.151

[Support and Size after Convolution]

Suppose that

the support of $x[n_1, n_2, \ldots, n_N]$ is $\{[n_1, n_2, \ldots, n_N]\ |n_1 \in (a_1, b_1), n_2 \in (a_2, b_2), \ldots.., n_N \in (a_N, b_N)\}$

the support of $h[n_1, n_2, \ldots, n_N[$ is $\{[n_1, n_2, \ldots, n_N]\ |n_1 \in (c_1, d_1), n_2 \in (c_2, d_2), \ldots.., n_N \in (c_N,\ d_N)\}$.

If

$$y[n_1,n_2,\cdots,n_N]=\sum_{k_N=-\infty}^{\infty}\cdots\sum_{k_2=-\infty}^{\infty}\sum_{k_1=-\infty}^{\infty}x[k_1,k_2,\cdots,k_N]h[n_1-k_1,n_2-k_2,\cdots,n_N-k_N]$$

the support of $y[n_1, n_2, \ldots, n_N]$ is

$\{[n_1, n_2, \ldots, n_N]\ |n_1 \in [a_1+c_1, b_1+d_1], n_2 \in [a_2+c_2, b_2+d_2],\ \ \ldots..,\ \ n_N \in [a_N+c_N, b_N+d_N]\}$.

the size of $y[n_1, n_2, \ldots, n_N]$ is

$(S_1 + T_1 - 1) \times (S_2 + T_2 - 1) \times \ldots. \times (S_N + T_N - 1)$

$(S_n = b_n - a_n+1)$
$(T_n = d_n - c_n+1)$

P.151

[Support and Size after Convolution]

For [Example 2.17]

the support of $x[n_1, n_2]$ $\{[n_1, n_2] \mid n_1 \in [-3, 3], n_2 \in [-3, 3]\}$

size: 7×7

the support of $h[n_1, n_2]$ $\{[n_1, n_2] \mid n_1 \in [-1, 1], n_2 \in [-1, 1]\}$

size: 3×3

the support of $y[n_1, n_2] = x[n_1, n_2] * h[n_1, n_2]$ is within

$$\{[n_1, n_2] \mid n_1 \in [-3\text{-}1, 3+1] = [-4, 4], n_2 \in [-3\text{-}1, 3+1] = [-4, 4]\}.$$

the size of $y[n_1, n_2]$

$$(7+3-1) \times (7+3-1) = 9 \times 9$$

P.151

Sec. 2.7 Several Well-known LTI Systems

Key concepts

Learning some well-known LTI systems, including
(i) difference and accumulation,
(ii) edge detection, and
(iii) smother and local average

P.152

- delay (continuous time)

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

- delay (discrete time)

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

- differentiation

$$x(t) * u_1(t) = \frac{d}{dt}x(t) \qquad x(t) * u_k(t) = \frac{d^k}{dt^k}x(t)$$

- difference

$$x[n] * (\delta[n] - \delta[n-1]) = x[n] - x[n-1]$$

P.152

- integral

$$x(t) * u_{-1}(t) = \int_{-\infty}^{t} x(\sigma)d\sigma$$

$$x(t) * u_{-k}(t) = \int_{-\infty}^{t}\int_{-\infty}^{\tau_{k-1}}\cdots\int_{-\infty}^{\tau_2}\left(\int_{-\infty}^{\tau_1} x(\sigma)d\sigma\right)d\tau_1 d\tau_2 \cdots d\tau_{k-1}$$

- accumulation

$$y[n] = \sum_{m=-\infty}^{n} x[m]$$

P.152-153

Edge Detection

$$x[n] * h[n]$$

where $h[n]$ satisfies

(i) $h[n] = -h[-n]$ for all n, i.e., $h[n]$ is odd,
(ii) $h[n] \to 0$ when $|n|$ is large,
(iii) $|h[n]|$ is larger when n is around zero.
$|h[n]|$ tends to decaying with $|n|$.

Example

$h_1[-1] = 1/\sqrt{2}$, $h_1[1] = -1/\sqrt{2}$,
$h_1[n] = 0$ otherwise,

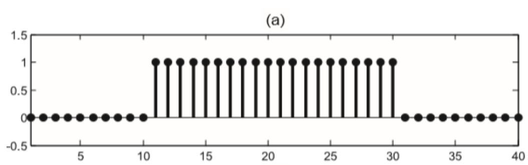
or

$h_2[1] = 8/\sqrt{260}$, $h_2[2] = 6/\sqrt{260}$, $h_2[3] = 4/\sqrt{260}$,
$h_2[4] = 3/\sqrt{260}$, $h_2[5] = 2/\sqrt{260}$, $h_2[6] = 1/\sqrt{260}$,
$h_2[n] = -h_2[-n]$ for $n = -1, -2, -3, -4, -5, -6$, $h_2[n] = 0$ otherwise.

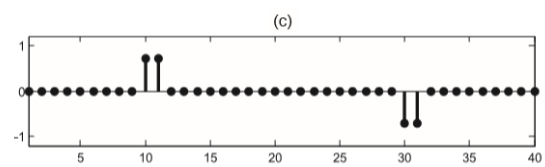
P.153

Edge Detection (short impulse response)

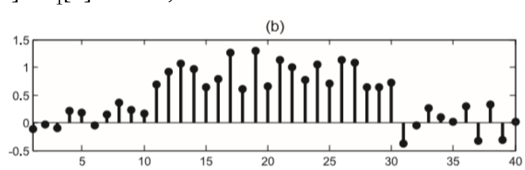
(a) A rectangular signal $x_1[n]$;



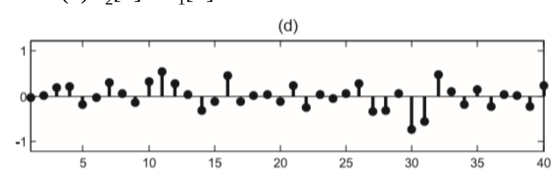
(c) $x_1[n] * h_1[n]$



(b) $x_2[n] = x_1[n]$ + noise;



(d) $x_2[n] * h_1[n]$

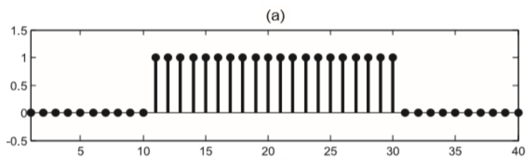


$h_1[-1] = 1/\sqrt{2}$, $h_1[1] = -1/\sqrt{2}$,
$h_1[n] = 0$ otherwise,

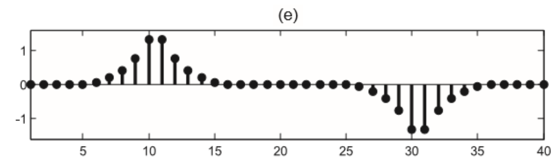
P.154

Edge Detection (long impulse response)

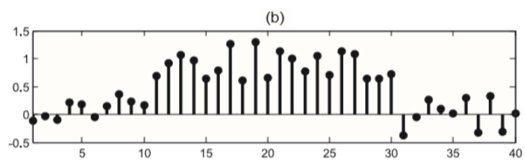
(a) A rectangular signal $x_1[n]$;



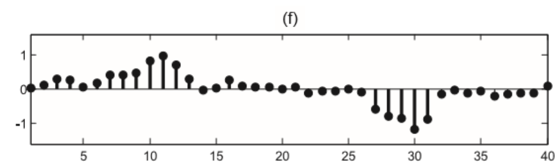
(e) $x_1[n] * h_2[n]$



(b) $x_2[n] = x_1[n]$ + noise;



(f) $x_2[n] * h_2[n]$



$$h_2[1] = 8/\sqrt{260}, \quad h_2[2] = 6/\sqrt{260}, \quad h_2[3] = 4/\sqrt{260},$$
$$h_2[4] = 3/\sqrt{260}, \quad h_2[5] = 2/\sqrt{260}, \quad h_2[6] = 1/\sqrt{260},$$
$$h_2[n] = -h_2[-n] \quad \text{for } n = -1, -2, -3, -4, -5, -6, \quad h_2[n] = 0 \text{ otherwise.}$$

P.154

Edge Detection (Two-Dimensional Case)

$$x[n_1, n_2] * h[n_1, n_2]$$

where $h[n_1, n_2]$ satisfies

(i) $h_1[n_1, n_2] = -h[-n_1, -n_2]$ for all n_1, n_2

(ii) $h_1[n_1, n_2] \to 0$ when $\sqrt{n_1^2 + n_2^2}$ is large,

(iii) $|h[n_1, n_2]| \geq |h[cn_1, cn_2]|$ if $c > 1$ and $[n_1, n_2] \neq [0, 0]$.

Example:

$$h[n_1, n_2] = \begin{bmatrix} 1 & 0 & -1 \\ a & 0 & -a \\ 1 & 0 & -1 \end{bmatrix} \qquad \text{for } -1 \leq n_1 \leq 1 \text{ and } -1 \leq n_2 \leq 1$$

P.155

Edge Detection (Two-Dimensional Case)

Example:

$$h[n_1,n_2]=\begin{bmatrix}1 & 0 & -1\\ a & 0 & -a\\ 1 & 0 & -1\end{bmatrix}$$

(horizontal edge detection)

$$h[n_1,n_2]=\begin{bmatrix}1 & a & 1\\ 0 & 0 & 0\\ -1 & -a & -1\end{bmatrix}$$

(vertical edge detection)

$$h[n_1,n_2]=\begin{bmatrix}0 & 1 & a\\ -1 & 0 & 1\\ -a & -1 & 0\end{bmatrix}$$

(45° edge detection)

$$h[n_1,n_2]=\begin{bmatrix}a & 1 & 0\\ 1 & 0 & -1\\ 0 & -1 & -a\end{bmatrix}$$

(135° edge detection)

P.155-156

Edge Detection (Two-Dimensional Case)

input

(a) $x[n_1, n_2]$

	$n_2=-4$	−3	−2	−1	0	1	2	3	4
$n_1=-4$	0	0	0	0	0	0	0	0	0
$n_1=-3$	0	255	255	255	255	255	255	255	0
$n_1=-2$	0	255	255	255	255	255	255	255	0
$n_1=-1$	0	255	255	255	255	255	255	255	0
$n_1=0$	0	255	255	255	255	255	255	255	0
$n_1=1$	0	255	255	255	255	255	255	255	0
$n_1=2$	0	255	255	255	255	255	255	255	0
$n_1=3$	0	255	255	255	255	255	255	255	0
$n_1=4$	0	0	0	0	0	0	0	0	0

(a)

output

	$n_2=-5$	−4	−3	−2	−1	0	1	2	3	4	5
$n_1=-5$	0	0	0	0	0	0	0	0	0	0	0
$n_1=-4$	0	255	255	0	0	0	0	0	−255	−255	0
$n_1=-3$	0	765	765	0	0	0	0	0	−765	−765	0
$n_1=-2$	0	1020	1020	0	0	0	0	0	−1020	−1020	0
$n_1=-1$	0	1020	1020	0	0	0	0	0	−1020	−1020	0
$n_1=0$	0	1020	1020	0	0	0	0	0	−1020	−1020	0
$n_1=1$	0	1020	1020	0	0	0	0	0	−1020	−1020	0
$n_1=2$	0	1020	1020	0	0	0	0	0	−1020	−1020	0
$n_1=3$	0	765	765	0	0	0	0	0	−765	−765	0
$n_1=4$	0	255	255	0	0	0	0	0	−255	−255	0
$n_1=5$	0	0	0	0	0	0	0	0	0	0	0

$$h[n_1,n_2]=\begin{bmatrix}1 & 0 & -1\\ 2 & 0 & -2\\ 1 & 0 & -1\end{bmatrix}$$

P.157

Edge Detection (Two-Dimensional Case)

input

(a) $x[n_1, n_2]$

	$n_2 = -4$	-3	-2	-1	0	1	2	3	4
$n_1 = -4$	0	0	0	0	0	0	0	0	0
$n_1 = -3$	0	255	255	255	255	255	255	255	0
$n_1 = -2$	0	255	255	255	255	255	255	255	0
$n_1 = -1$	0	255	255	255	255	255	255	255	0
$n_1 = 0$	0	255	255	255	255	255	255	255	0
$n_1 = 1$	0	255	255	255	255	255	255	255	0
$n_1 = 2$	0	255	255	255	255	255	255	255	0
$n_1 = 3$	0	255	255	255	255	255	255	255	0
$n_1 = 4$	0	0	0	0	0	0	0	0	0

output

(b)

	$n_2 = -5$	-4	-3	-2	-1	0	1	2	3	4	5
$n_1 = -5$	0	0	0	0	0	0	0	0	0	0	0
$n_1 = -4$	0	255	765	1020	1020	1020	1020	1020	765	255	0
$n_1 = -3$	0	255	765	1020	1020	1020	1020	1020	765	255	0
$n_1 = -2$	0	0	0	0	0	0	0	0	0	0	0
$n_1 = -1$	0	0	0	0	0	0	0	0	0	0	0
$n_1 = 0$	0	0	0	0	0	0	0	0	0	0	0
$n_1 = 1$	0	0	0	0	0	0	0	0	0	0	0
$n_1 = 2$	0	0	0	0	0	0	0	0	0	0	0
$n_1 = 3$	0	−255	−765	−1020	−1020	−1020	−1020	−1020	−765	−255	0
$n_1 = 4$	0	−255	−765	−1020	−1020	−1020	−1020	−1020	−765	−255	0
$n_1 = 5$	0	0	0	0	0	0	0	0	0	0	0

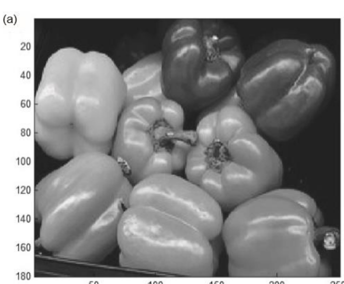
$$h[n_1, n_2] = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

P.157

Edge Detection (Two-Dimensional Case)

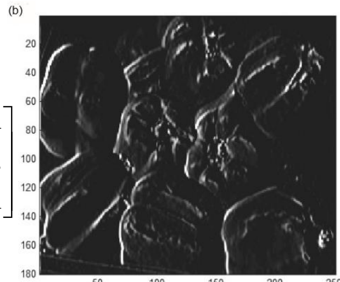
input

(a)



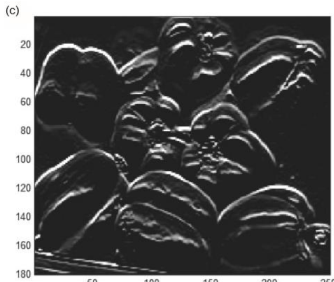
output

(b)



$$h[n_1, n_2] = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

(c)



output

$$h[n_1, n_2] = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

P.158

Smoother

$$y[n] = x[n] * h[n]$$

(i) $h[n] = h[-n]$, (i.e., $h[n]$ is even),
(ii) $h[n] \to 0$ when $|n|$ is large,
(iii) $|h[n_1]| \geq |h[n_2]|$ if $|n_1| < |n_2|$,
(iv) $\sum_{n=-\infty}^{\infty} h[n] = 1$,
(v) $h[n] \geq 0$ for all n.

Example

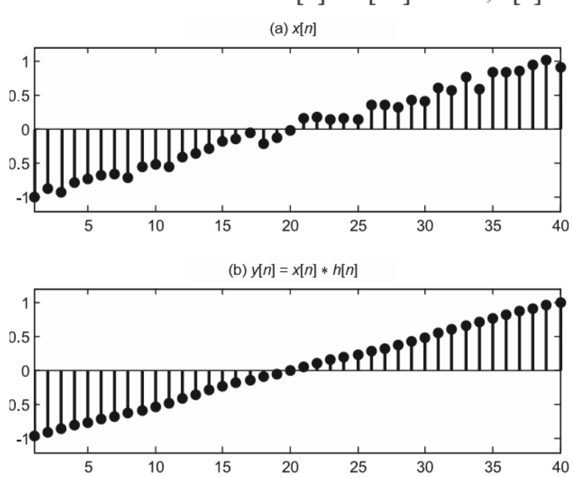
$$h[n] = \frac{1}{2L_1+1} \quad \text{for } -L_1 \leq n \leq L_1, \qquad h[n] = 0 \quad \text{otherwise}$$

$$y[n] = x[n] * h[n] = \frac{1}{2L_1+1} \sum_{m=n-L_1}^{n+L_1} x[m] \qquad \text{(local average)}$$

P.157-158

Smoother

Example $h[0] = 0.2$, $h[1] = h[-1] = 0.16$, $h[2] = h[-2] = 0.12$, $h[3] = h[-3] = 0.08$, $h[4] = h[-4] = 0.04$, $h[n] = 0$ otherwise.



$$x[n] = 0.05(n-20) + noise$$

P.159

Sec. 2.8 Summary

- In this chapter, we have developed the representations for LTI systems by convolution operations, both in discrete time and in continuous time.

- LTI systems can be analyzed by the Fourier series (FS), the Fourier transform (FT), the Laplace transform (LT), and the z-transform (ZT).

Periodic

FS (Chap 3) — CT, DT

Aperiodic

FT — CT (Chap 4), DT (Chap 5)

Unbounded/Non-convergent

LT — CT (Chap 9)

zT — DT (Chap 10)

Sec. 2.9 Further Reading

Matlab for convolution

$$y = \text{conv}(x, h).$$

$$y = \text{conv2}(x, h), \qquad y = \text{convn}(x, h).$$

P.160

Processing an audio file

[x, fs] = audioread('filename'); % read

audiowrite('filename', x, fs); % create an audio file

sound(x, fs); % play

x: a column vector
or two column vectors (stereophonic case)

P.161

Processing an image file

x = double(imread('filename')); % read

imwrite(x, 'filename'); % create an image file

image(x); or imagesc(x); % display

image(x); colormap(gray(256)); % display a gray-level image

x: a matrix (gray level image)
or three matrices (color image)

P.161

Processing a video file

```
OBJ  =  VideoReader('****.mp4');          % read
 x = read(OBJ);
x = double(x);

vidObj = VideoWriter('test.avi');   % create a video file
 open(vidObj);
writeVideo(vidObj, x);
close(vidObj)

implay('filename');     or  implay(x, nf);     display
```

P.161