

Signal and System Midterm

2016/01/11

班級：

姓名：

學號：

共八題，配分共 140 分，最高分以 100 分計

1. (20%) Use the Fourier transform analysis equation to calculate the Fourier transforms of the following signals:

$$(a) x_1(t) = \delta(t + 3) + \delta(t - 3) \quad (b) x_2(t) = \frac{d}{dt} \{u(-5 - t) + u(-5 + t)\}$$

$$(c) x_3[n] = \left(\frac{1}{3}\right)^{n-1} u[n - 1] \quad (d) x_4[n] = \left(\frac{1}{5}\right)^{|n-1|}$$

2. (20%) Determine the Fourier transform in the case of each of the following periodic signals:

$$(a) x_1[n] = \sin\left(\frac{\pi}{4}n + \frac{\pi}{6}\right), \text{ for } -\pi \leq \omega < \pi \quad (b) x_2[n] = 2 + \cos\left(\frac{\pi}{3} + \frac{\pi}{6}\right), \text{ for } -\pi \leq \omega < \pi$$

$$(c) x_3(t) = \sin\left(2\pi t + \frac{\pi}{4}\right) \quad (d) x_4(t) = 1 + \cos\left(6\pi t + \frac{\pi}{8}\right)$$

3. (10%) (a) Determine the Fourier transform of the following signal:

$$x(t) = t \left(\frac{\sin t}{\pi t}\right)^2$$

(10%) (b) Use Parseval's relation and the result of the previous part to determine the numerical value of

$$A = \int_{-\infty}^{+\infty} t^2 \left(\frac{\sin t}{\pi t}\right)^4 dt$$

4. (20%) Given the relationships

$$y(t) = x(t) * h(t)$$

and

$$g(t) = x(3t) * h(3t)$$

and given that $x(t)$ has Fourier transform $X(j\omega)$ and $h(t)$ has Fourier transform $H(j\omega)$, use Fourier transform properties to show that $g(t)$ has the form $g(t) = Ay(Bt)$. Determine the values of A and B .

5. (10%) Determine the periodic signal for $-\pi \leq \omega < \pi$, whose Fourier transform is:

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \{2\pi\delta(\omega - 2\pi k) + \pi\delta\left(\omega - \frac{\pi}{2} - 2\pi k\right) + \pi\delta\left(\omega + \frac{\pi}{2} - 2\pi k\right)\}$$

6. (20%) Consider a causal and stable LTI system \mathbf{S} , its input $x[n]$ and the corresponding output $y[n]$ is
- $$y[n + 1] - 3y[n] + 2y[n - 1] = x[n]$$
- (a) Find the frequency response $H(e^{j\omega})$ of the system \mathbf{S} .
- (b) Find the impulse response $h[n]$ of the system \mathbf{S} .
7. (10%) Determine the Nyquist rate corresponding to each of the following signals:
- (a) $x_1(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$
- (b) $x_2(t) = \frac{\sin(4000\pi t)}{\pi t}$
8. (20%) Let $x(t)$ be a signal with Nyquist rate ω_0 . Determine the Nyquist rate for each of the following signals:
- (a) $x(t) + x(t - 1)$
- (b) $\frac{dx(t)}{dt}$
- (c) $x^2(t)$
- (d) $x(t)\cos\omega_0 t$