Signals and Systems Final Examination

Name: ____________________________ Student ID: ____________________________ 94/01/13

1. (5%) Determine the fundamental period of the discrete-time signal $x[n] = e^{j(2\pi/5)n} - e^{j(3\pi/4)n}$.

2. (10%) Please determine the signal $y(t)$ as the convolution of the following two signals: $x(t) = e^{3t}u(-t)$ and $h(t) = u(t-2)$.

3. (10%) Consider the signal $x(t) = 2[1 + \cos w_0 t + \sin(2w_0 t + \frac{\pi}{4})]$, whose fundamental signal is w_0. Please determine the corresponding Fourier series coefficients.

4. (10%) Consider that a discrete-time signal $x[n]$ is periodic with period L, the fundamental frequency w_0 becomes ___________. If a_k is the Fourier series coefficients, the discrete-time Fourier series representation of $x[n]$ can be expressed as

$$x[n] = \underline{\hspace{6cm}},$$

where a_k can be determined from $x[n]$ by the use of the equation

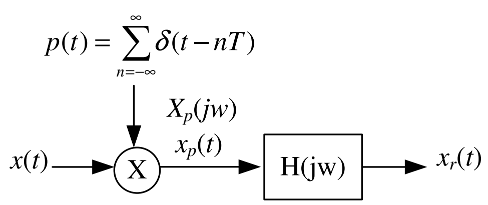
$$a_k = \underline{\hspace{6cm}}.$$

5. (10%) Consider the signal

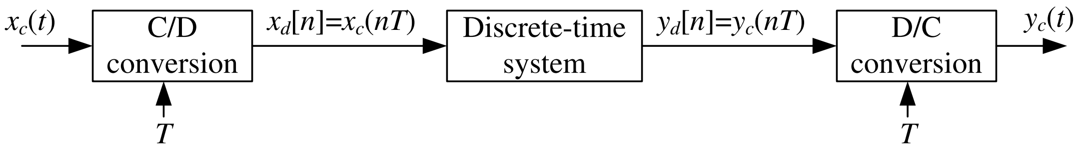
$$x[n] = \sin w_0 n = \frac{1}{2j}(e^{jw_0 n} - e^{-jw_0 n}), \text{ with } w_0 = \frac{2\pi}{5}.$$

Please determine the Fourier transform of $x[n]$ and depict $X(e^{jw})$ within one period.

6. (16%) Consider the figure shown below for a system for sampling a band-limited signal $x(t)$ ($X(jw) = 0$ for $|w| > w_M$) and reconstruction. Let w_s denote the sampling frequency. That is, $w_s = 2\pi/T$. Please draw the spectra of following signals under two conditions - $w_s > 2w_M$ and $w_s < 2w_M$: (a) representative spectrum for $x(t)$; (b) corresponding spectrum for $x_p(t)$; (c) ideal lowpass filter to recover $X(jw)$ from $X_p(jw)$; (d) spectrum of $x_r(t)$.



7. (20%) The continuous-time Fourier transforms of $x_c(t)$ and $y_c(t)$ are $X_c(jw)$ and $Y_c(jw)$, respectively, while the discrete-time Fourier transform of $x_d[n]$ and $y_d[n]$ are $X_d(e^{j\Omega})$ and $Y_d(e^{j\Omega})$, respectively. The relationships between these signals are shown in the figure below.



Since $x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\delta(t-nT)$ and since the transform of $\delta(t-nT)$ is $\underline{\qquad\qquad\qquad}$, it follows that

$$X_p(jw) = \sum_{n=-\infty}^{\infty} \underline{\qquad\qquad\qquad}, X_d(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_d[n]\underline{\qquad\qquad\qquad} = \sum_{n=-\infty}^{\infty} x_c(nT)\underline{\qquad\qquad\qquad}$$

Note that $X_d(e^{j\Omega})$ and $X_p(jw)$ are related through $X_d(e^{j\Omega}) = X_p(\underline{\qquad\qquad\qquad})$ and

$$X_p(jw) = \frac{1}{T}X_c(\underline{\qquad\qquad\qquad}), X_d(e^{j\Omega}) = \frac{1}{T}X_c(\underline{\qquad\qquad\qquad})$$

Therefore, $X_d(e^{j\Omega})$ is a $\underline{\qquad\qquad\qquad}$ version of $X_p(jw)$.

8. (10%) Use Tables 4.1 and 4.2 to help determine the Fourier transform of the following signal:

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$$

9. (10%) Use Tables 5.1 and 5.2 to help determine $x[n]$ when its Fourier transform is

$$X(e^{jw}) = \frac{1}{1-e^{-jw}}\left(\frac{\sin\frac{3}{2}w}{\sin\frac{w}{2}}\right) + 5\pi\delta(w), \quad -\pi < w \leq \pi.$$

10. (5%) Determine the *Nyquist rate* corresponding the following signal:

$$x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t).$$

11. (8%) Let $x[n]$ be a real and odd periodic signal with period $N = 7$ and Fourier coefficients a_k. Given that $a_{15} = j$, $a_{16} = 2j$, $a_{17} = 3j$, determine the values of a_0, a_{-1}, a_{-2}, and a_{-3}.

12. (5%) The definition of a sinc function is $\text{sinc}(\theta) = \frac{\sin \pi\theta}{\pi\theta}$. Please rewrite the following signal in terms of the sinc functions :

$$\frac{\sin 2Wt}{\pi t} = \underline{\hspace{6cm}}.$$