Signal and System Midterm
2015/11/16

班級 :　　　　　　　　姓名 :　　　　　　　　學號 :

共十題，滿分 100 分

1. A discrete-time signal $x[n]$ is shown in Fig. 1. Sketch and label carefully each of the following signal:

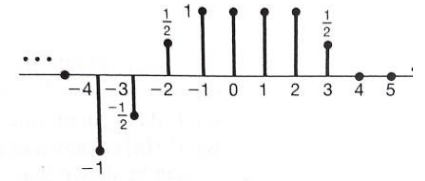   (a) $x[3n + 4]$　　　　(b) $x[(n - 1)^2]$

Figure 1

2. Determine the value of P_∞ and E_∞ for each of the following signals:

    (a) $x_1(t) = e^{-j(3t+\pi/6)}$　　　　(b) $x_2(t) = \sin(t)$

    (c) $x_3[n] = \left(\frac{1}{3}\right)^n u(n)$　　　　(d) $x_4[n] = \sin\left(\frac{\pi}{4}n\right)$

3. Suppose that

    $$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

    and $h(t) = x(t/\alpha)$, where $0 \le \alpha \le 1$.

    (a) Determine and sketch $y(t) = x(t) * h(t)$.

    (b) if $\frac{dy(t)}{dt}$ contains only two discontinuities, what is the value of α?

4. Compute the convolution of the following pairs of signal:

    (a) $\left.\begin{array}{l} x[n] = \alpha^n u[n], \\ h[n] = \beta^n u[n], \end{array}\right\} \alpha \ne \beta$

    (b) $\left.\begin{array}{l} x(t) = e^{-\alpha t} u(t), \\ h(t) = e^{-\beta t} u(t), \end{array}\right\} \alpha \ne \beta$

5. Draw block diagram representations for causal LTI system described by the following equations:

    (a) $y[n] = 3 y[n - 2] + 4y[n - 1] + 5 x[n]$

    (b) $\frac{dy(t)}{dt} = 5x(t) + 3y(t)$

6. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 6$. The nonzero-Fourier series coefficients for $x(t)$ are

    $$a_1 = a_{-1}^* = 2, \quad a_2 = a_{-2}^* = -3j.$$

    Express $x(t)$ in the form

    $$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$

7. Using the fundamental frequency $\omega_0 = \pi$ to calculate the Fourier series coefficients a_k of the continuous-time periodic signal $x(t)$:

$$x(t) = \begin{cases} 2, & 0 \leq t < 1 \\ -2, & 1 \leq t < 2 \end{cases}$$

8. Consider a continuous-time LTI system whose frequency response is

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t} dt = \frac{\sin(4\omega)}{\omega}.$$

if input $x(t)$ is periodic with period $T=8$

$$x(t) = \begin{cases} 1, & 0 \leq t < 4 \\ -1, & 4 \leq t < 8' \end{cases}$$

find the output $y(t)$ of this system

9. Suppose we are given the following information about a signal $x(t)$:

(1) $x(t)$ is real and odd.

(2) $x(t)$ is periodic with period $T=3$ and has Fourier coefficients a_k.

(3) $a_k = 0$ for $|k| > 1$.

(4) $\frac{1}{3}\int_0^3 |x(t)|^2 dt = 1$.

Specify two different signals that satisfy these conditions.

10. Suppose we are given the following information about a signal $x[n]$:

(1) $x[n]$ is a real and even signal.

(2) $x[n]$ has period $N=10$ and Fourier coefficients a_k.

(3) $a_{11} = 5$.

(4) $\frac{1}{10}\sum_{n=0}^{9} |x[n]|^2 = 50$

Show that $x[n] = A\cos(Bn + C)$, and specify numerical values for the constants A, B, and C.