Signals and Systems Quiz #4

Name: _____   ID No.: _____   93/11/25

1. (30%) Suppose that $x(t)$ is real and can be represented as the form of

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}.$$

As shown in Eq. (3.30), $x(t)$ can be also represented as the form of

$$x(t) = a_0 + \sum_{k=1}^{\infty} 2\mathcal{R}e\{a_k e^{jkw_0 t}\}.$$

By writing a_k in rectangular form as $a_k = B_k + jC_k$, show that $x(t)$ can be represented as the form

$$x(t) = a_0 + 2\sum_{k=1}^{\infty}[B_k \cos kw_0 t - C_k \sin kw_0 t].$$

2. (10%) The coefficient of the Fourier series of a real signal $x(t)$ with period T is defined as

$a_k =$ _____.

3. (30%) Consider the signal $x(t) = 2[1 + \cos w_0 t + \sin(2w_0 t + \frac{\pi}{4})]$, whose fundamental signal is w_0. Please determine the corresponding Fourier series coefficients.

4. (10%) When a periodic signal is shifted in time, the _____ of its Fourier series coefficients remain unaltered.

5. (30%) Consider the three Dirichlet conditions: (1) Over any period, $x(t)$ must be absolutely integrable; (2) In any finite interval of time, $x(t)$ is of bounded variation; (3) In any finite interval of time, there are only a finite number of discontinuities. Furthermore, each of these discontinuities is finite. For the signal

$$x(t) = \cos(\frac{2\pi}{t}), \quad 0 < t \le 1,$$

which condition does $x(t)$ violate? Please also give brief explanation on your decision.