Signals and Systems Quiz #5 (Sec. 3.5–3.9)

Name: ______________________ ID No.: ______________________ 93/12/8

1. (10%) Suppose that $x(t)$ and $y(t)$ are both periodic with period T and that $x(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} a_k$ and $y(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} b_k$. Then

$$x(t)y(t) \stackrel{\mathcal{FS}}{\longleftrightarrow} h_k = \sum_{l=-\infty}^{\infty} \underline{\hspace{4cm}}.$$

2. (10%) What operation in general changes the period (fundamental frequency) of the underlying signal? (a) Time shifting; (b) Time reversal; (c) Time scaling; (d) Multiplication; (e) Conjugation and conjugate symmetry.
Answer: __________

3. (10%) For the real and even periodic signal $x(t)$, its Fourier series coefficients a_k are also real and even. If the periodic signal $x(t)$ is real and odd, on the other hand, the corresponding Fourier series coefficients a_k are purely ______________ and ______________.

4. (30%) Consider that a discrete-time signal $x[n]$ is periodic with period L, the fundamental frequency w_0 becomes __________. If a_k is the Fourier series coefficients, the discrete-time Fourier series representation of $x[n]$ can be expressed as

$$x[n] = \underline{\hspace{7cm}},$$

where a_k can be determined from $x[n]$ by the use of the equation

$$a_k = \frac{1}{L}\underline{\hspace{7cm}}.$$

5. (10%) Suppose that $x[n]$ and $y[n]$ are both periodic with period N and that $x[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} a_k$ and $y[n] \stackrel{\mathcal{FS}}{\longleftrightarrow} b_k$. Then the Fourier series coefficients c_k of the periodic convolution of $x[n]$ and $y[n]$ are equal to

$$\sum_{r=<N>} x[r]y[n-r] \stackrel{\mathcal{FS}}{\longleftrightarrow} c_k = \underline{\hspace{4cm}}.$$

6. (20%) Let $x(t)$ be a periodic signal with a Fourier series representation given by

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0t}.$$

Suppose that this signal is applied to an LTI system with impulse response $h(t)$. The output $y(t)$ can be represented as

$$y(t) = \underline{\sum_{k=-\infty}^{\infty} \hspace{4cm}}.$$

The set of Fourier series coefficients for the output $y(t)$ is $\underline{\{\hspace{4cm}\}}$.

7. (10%) Consider the filter output is the derivative of the filter input, i.e., $y(t) = dx(t)/dt$. If the input $x(t) = e^{jwt}$, then the frequency response is $H(jw) =$ __________, which is a frequency-shaping or frequency-selective filter? Ans: __________.

8. (20%) The frequency response of a continuous-time ideal lowpass filter is

$$H(jw) = \begin{cases} 1, & |w| \le w_c \\ 0, & |w| > w_c \end{cases}.$$

(1) The parameters w_c is called the __________ frequeycy.
(2) Please draw the frequency response of the lowpass filter based on the equation above.