Signals and Systems Midterm Examination

Name: _____ ID No.: _____ 93/11/11

1. (10%) A signal for which the system output is constant times the input is referred to as an _____ of the system, and the amplitude factor is referred to as the system's _____.

2. (10%) In addition to the impulse response, $h[n]$ or $h(t)$, the unit step response, $s[n]$ or $s(t)$, is also used quite often in describing the behavior of LTI systems. The definitions of the unit step response for discrete- and continuous-time systems are $s[n] = u[n] * h[n]$ and $s(t) = u(t) * h(t)$, respectively. What are the relations that $h[n]$ and $h(t)$ can be recovered from $s[n]$ or $s(t)$?

3. (10%) For an input $x(t) = e^{st}$, we can determine the output through the use of the convolution integral, i.e.,

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = H(s)e^{st}.$$

Please show that

$$H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau.$$

4. (10%) In Problem 3, if $y(t) = x(t-2)+x(t-3)$, that is, the impulse response of system is $h(t) = \delta(t-2)+\delta(t-3)$, please determine the function $H(s)$.

5. (10%) (a) Consider an LTI system with input and output related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau-2)d\tau.$$

What is the impulse response $h(t)$ for this system?
(b) Determine the response of the system when the input is $x(t) = u(t+1) - u(t-2)$.

6. (10%) Let

$$y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k).$$

Show that $y(t) = Ae^{-t}$ for $0 \le t < 3$, and determine that value of A.

7. (20%) An important concept in many communications applications is the *correlation* between two signals. Let $x(t)$ and $y(t)$ be two signals; then the *correlation function* is defined as

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)y(\tau)d\tau.$$

The function $\phi_{xx}(t)$ is usually referred to as the *autocorrelation function* of the signal $x(t)$, while $\phi_{xy}(t)$ is called a *cross-correlation function.*
(a) What is the relationship between $\phi_{xy}(t)$ and $\phi_{yx}(t)$?
(b) Compute the odd part of $\phi_{xx}(t)$.

8. (10%) For each of the following two input-output relationships, determine whether the corresponding system is linear, time invariant or both. (a) $y(t) = t^2x(t-1)$; (b) $y[n] = x^2[n-2]$.

9. (10%) Determine whether or not each of the following continuous-time and discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.
(a) $x(t) = 3\cos(4t + \frac{\pi}{3})$; (b) $x[n] = \sin(\frac{6\pi}{7}n + 1)$.

10. (10%) Which of the following impulse response correspond(s) to stable LTI systems?
(a) $h_1(t) = e^{-(1-2j)t}u(t)$; (b) $h_4[n] = 3^n u[-n+10]$.

11. (10%) For the continuous-time periodic signal

$$x(t) = 2 + 3\cos(\frac{2\pi}{3})t + 4\sin(\frac{5\pi}{3}t),$$

determine the fundamental frequency w_0 and the Fourier series coefficients a_k such that

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}.$$