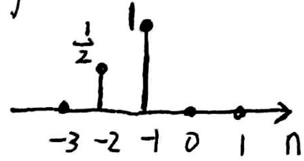
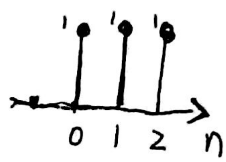
1、(a)



(b)



2、(a) $P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_1(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} dt = 1$

$E_\infty = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_{-\infty}^{\infty} dt = \infty$

(b) $P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x_2(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left(\frac{1-\cos 2t}{2}\right) dt = \frac{1}{2}$

$E_\infty = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{-\infty}^{\infty} \left(\frac{1-\cos 2t}{2}\right) dt = \infty$

(c)

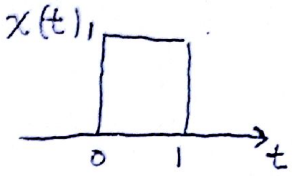
$E_\infty = \sum_{n=-\infty}^{\infty} |x_3[n]|^2 = \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n = \frac{9}{8}$

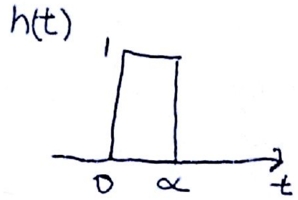
$\because E_\infty < \infty \quad \therefore P_\infty = 0$

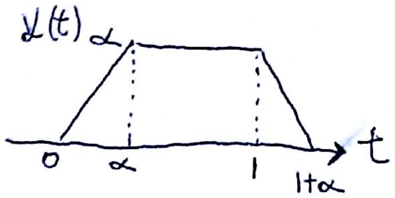
(d) $E_\infty = \sum_{n=-\infty}^{\infty} |x_4[n]|^2 = \sum_{n=-\infty}^{\infty} \sin^2\left(\frac{\pi}{4}n\right) = \infty$

$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \frac{1-\cos\left(\frac{\pi}{2}n\right)}{2} = \frac{1}{2}$

3 (a)







$$\Rightarrow \quad y(t)=\begin{cases} t, & \longrightarrow 0\le t\le\alpha \\ \alpha, & \longrightarrow \alpha\le t\le 1 \\ 1+\alpha-t, & \longrightarrow 1\le t\le (1+\alpha) \\ 0, & \longrightarrow \text{otherwise} \end{cases}$$

(b) 從圖中可發現，不連續點在 0，α，1，和 $1+\alpha$

∴只有在 $\alpha=1$ 時符合

4.

(a) $y[n]=x[n]*h[n]$

$$=\sum_{K=-\infty}^{\infty} x[K]\,h[n-K]$$

$$=\beta^n\sum_{K=0}^{n}\left(\frac{\alpha}{\beta}\right)^K \text{ for } n\ge 0$$

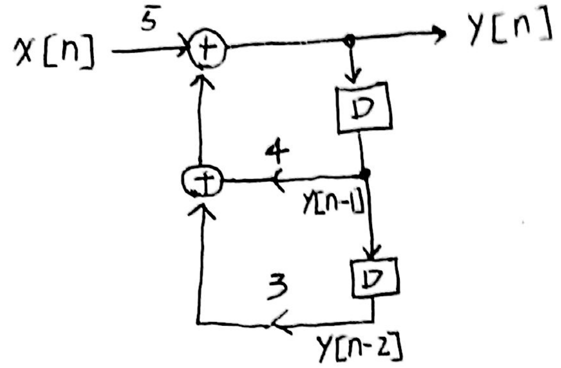
$$=\left[\frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha}\right]u[n] \text{ for } \alpha\ne\beta$$

(b) $y(t)=\int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$

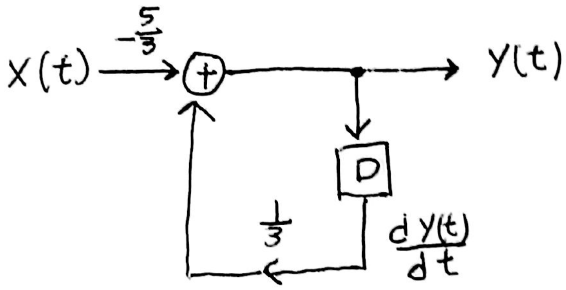
$$=\int_0^t e^{-\alpha t}e^{-\beta(t-\tau)}\,d\tau,\ t\ge 0$$

$$=\frac{e^{-\beta t}\{e^{-(\alpha-\beta)t}-1\}}{\beta-\alpha}u(t)$$

5 (a)



(b)



$\rightarrow y(t) = \frac{1}{3}\frac{dy(t)}{dt} - \frac{5}{3}x(t)$

6.

$$x(t) = 2\left(e^{j(2\pi/T)t} + e^{-j(2\pi/T)t}\right) - 3j\left(e^{j3(2\pi/T)t} + e^{-j3(2\pi/T)t}\right.$$

$$= 4\cos\left(\frac{2\pi}{T}t\right) + 6\sin\left(\frac{6\pi}{T}t\right)$$

$$= 4\cos\left(\frac{\pi}{3}t\right) + 6\cos\left(\pi \cdot t + \frac{\pi}{2}\right)$$

7

$$W_0 = \pi \Rightarrow T = \frac{2\pi}{W_0} = 2$$

$$a_k = \frac{1}{2}\int_0^2 x(t)e^{-jk\pi t}\,dt$$

$$k=0 \rightarrow a_0 = \frac{1}{2}\int_0^1 2\,dt - \frac{1}{2}\int_1^2 2\,dt = 0$$

$$k \neq 0 \rightarrow a_k = \frac{1}{2}\int_0^1 2e^{-jk\pi t}\,dt - \frac{1}{2}\int_1^2 2e^{-jk\pi t}\,dt$$

$$= \left(\frac{e^{-jk\pi t}}{-k\pi j}\right)\Bigg|_0^1 - \left(\frac{e^{-jk\pi t}}{-jk\pi}\right)\Bigg|_1^2$$

$$= \frac{1}{-jk\pi}\left(e^{-jk\pi} - 1 - e^{-jk(2\pi)} + e^{-jk\pi}\right.$$

$$= \frac{1}{jk\pi}\left(e^{-jk\pi} - 1\right)^2$$

8

$\because$ $x(t)$ is real and odd

$\therefore$ a_k is purely imaginary and odd $\Rightarrow a_0 = 0$

$$a_k = \frac{1}{8}\int_0^8 x(t) e^{-j(\frac{2\pi}{8})kt} dt$$

$$= \frac{1}{8}\int_0^4 e^{-j(2\pi/8)kt} dt - \frac{1}{8}\int_4^8 e^{-j(2\pi/8)kt} dt$$

$$= \frac{1}{j\pi k}[1 - e^{-j\pi k}]$$

$$\Rightarrow a_k = \begin{cases} 0 \longrightarrow k = 0, \pm 2, \pm 4 \cdots \\ \frac{2}{jk\pi} \longrightarrow k = \pm 1, \pm 3, \pm 5 \cdots \end{cases}$$

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$$

$\because$ $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{4} \longrightarrow H(jk\omega_0) = H(jk(\frac{\pi}{4}))$

$$= \frac{\sin(k\pi)}{k(\pi/4)}$$

$\because$ $k = \pm 1 \ \pm 3 \ \pm 5 \cdots$ $H(jk\omega_0) = 0$

$\therefore$ $y(t) = 0$ #

9. $x(t) \rightarrow$ real and odd

$a_k \Rightarrow$ purely imaginary and odd

$(a_k = -a_{-k},\ a_0 = 0)$

$\because$ $a_k = 0$ for $|k| > 1$

$\therefore$ 只有 a_1 和 a_{-1} 未知0，其餘 $= 0$

parseval's relation $\rightarrow \frac{1}{T}\int_{<T>} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$

$$\Rightarrow \frac{1}{3}\int_0^3 |x(t)|^2 dt = \sum_{k=-1}^{1} |a_k|^2$$

$$\rightarrow |a_1|^2 + |a_{-1}|^2 = 1 \rightarrow 2|a_1|^2 = 1$$

$$\Rightarrow a_1 = -a_{-1} = \pm\frac{1}{\sqrt{2}j}$$

$$\Rightarrow x_1(t) = \frac{1}{\sqrt{2}j} e^{j(2\pi/3)t} - \frac{1}{\sqrt{2}j} e^{-j(2\pi/3)t} = -\sqrt{2}\sin(\frac{2\pi}{3}t)$$

$$x_2(t) = -\frac{1}{\sqrt{2}j} e^{j(2\pi/3)t} + \frac{1}{\sqrt{2}j} e^{-j(2\pi/3)t} = \sqrt{2}\sin(\frac{2\pi}{3}t)$$

10、 x[n]: real and even $\Rightarrow$ a_k : real and even

$$\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50$$

$a_{11} = a_1 = 5$

$a_1 = a_{-1} = 5$

Parsevel's relation

$$\Rightarrow \sum_{k=\langle N\rangle} |a_k|^2 = 50$$

$$\sum_{k=-1}^{8} |a_k|^2 = 50$$

$$|a_{-1}|^2 + |a_0|^2 + |a_1|^2 + \sum_{k=2}^{8} |a_k|^2 = 50$$

$$|a_0|^2 + \sum_{k=2}^{8} |a_k|^2 = 0$$

$\therefore a_k = 0$ for $k = 2, 3, \dots 8$

$$x[n] = 5e^{j\frac{2\pi}{10}n} + 5e^{-j\frac{2\pi}{10}n}$$

$$= 10\cos\left(\frac{\pi}{5}n\right)$$

$\therefore A = 10$, $B = \frac{\pi}{5}$, $C = 0$